Physics 21 W'11

Close Out of Physics 20

Final: Avg = 72/100

$\sigma = 18$

13 out of 90 > 90/100

1 perfect score.

Overall: Avg 70/100 ≈ B

$\sigma = 19$

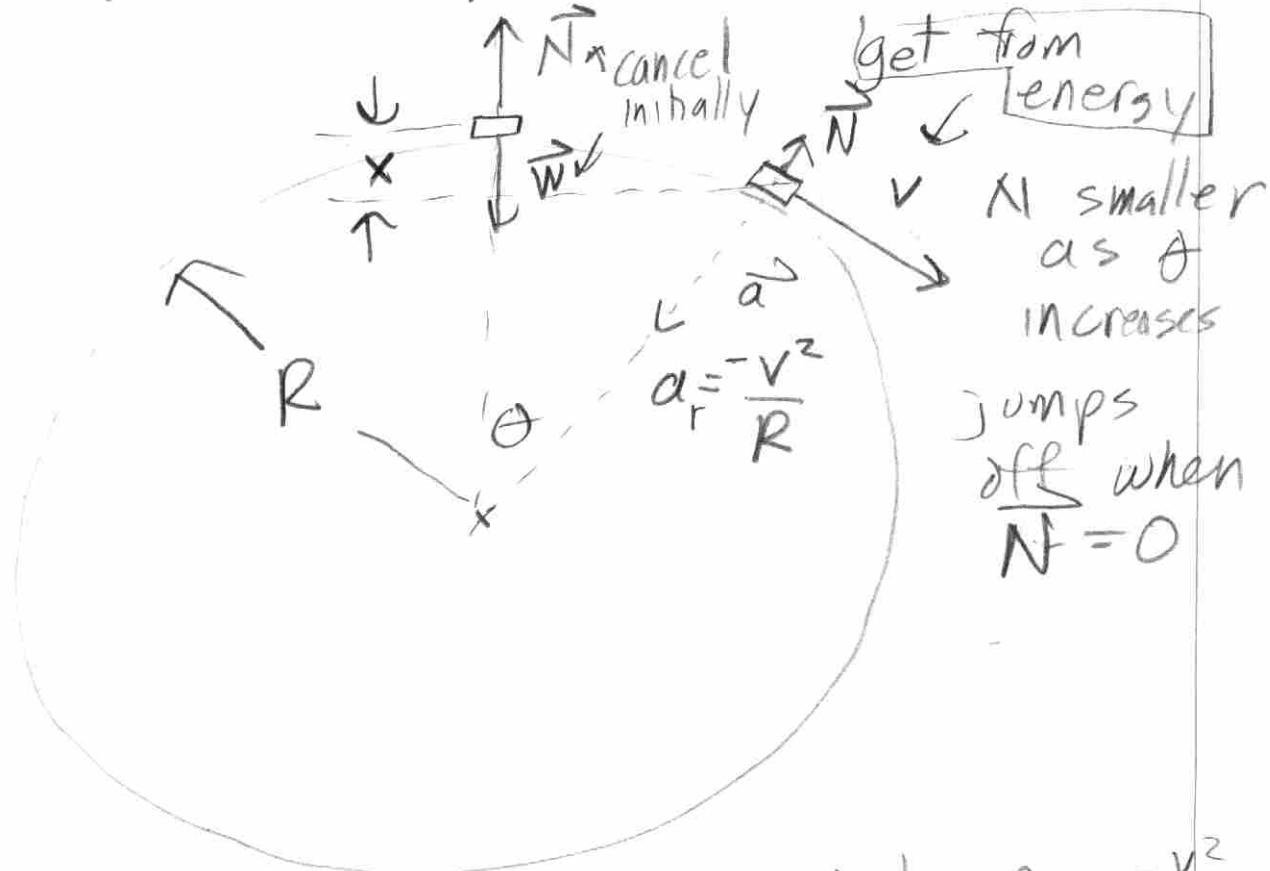
A− > 84

B− > 66

C− > 45

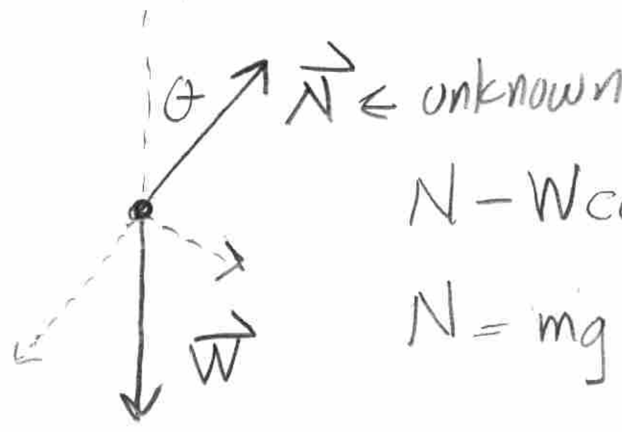
Final problem ...

A small block slides from rest from the top of a frictionless sphere of radius R. How far from the top x does it lose contact with the sphere? Sphere is immobile



$$\sum \vec{F} = m\vec{a} \;\Big]\; \begin{array}{l} \bullet \text{ radial } \quad a_r = \frac{-v^2}{R} \\ \bullet \text{ tangential } \quad a_t \geq 0 \\ \quad\quad \text{weight..} \end{array}$$

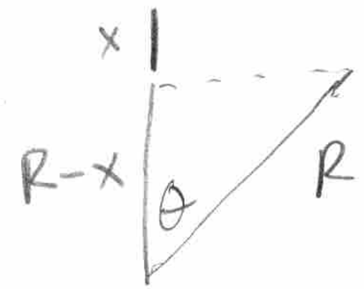
N gets smaller as θ increases



$$N - W\cos\theta = -m\frac{v^2}{R}$$

$$N = mg\cos\theta - m\frac{v^2}{R}$$

$\cos\theta \rightarrow$ replace with x



$$\cos\theta = \frac{R-x}{R} = 1 - \frac{x}{R}$$

Energy for v:

Initial	Final
$\underbrace{\frac{1}{2}m\cdot 0^2}_{\text{Kinetic}} + \underbrace{0}_{\text{potential}}$ $=$	$\frac{1}{2}mv^2 - mgx$

$$0 = \frac{1}{2}\cancel{m}v^2 - \cancel{m}gx$$

$$\text{or } v^2 = 2gx$$

$$N = 0 = \cancel{m}\cancel{g}\left(1 - \frac{x}{R}\right) - \cancel{m}\cdot\frac{2\cancel{g}x}{R}$$

$$= 1 - \frac{3x}{R}$$

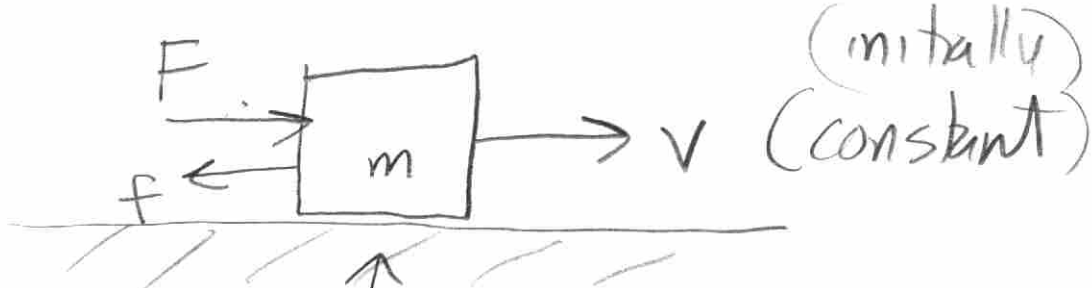
$$\boxed{x = \frac{R}{3}}$$

Think about trajectory ...

Power

is work done per unit time...

2 Flavors :
- average
- instantaneous



kinetic friction μ_K

y

$\vec{N}$

$$\vec{N} = mg\hat{j}$$

$$\vec{W} = -mg\hat{j}$$

$$f = \mu_K N = \mu_K mg$$

v constant: $\quad a = 0$

$$\Sigma F_x = 0$$

$$F - f = 0$$

$$F = f = \mu_K mg$$

$t = 0$ at $x = 0$

$t = t$ at.. $x = vt$

Work : $W = \int \underset{\substack{\uparrow \\ \text{constant}}}{F}\,dx = F \cdot x = Fvt$

$$W = Fvt$$

AVERAGE POWER ...

$$\overline{P} \equiv \frac{W}{t} = \frac{Fvt}{t} = Fv$$

Units : watt $\equiv \dfrac{\text{Joules}}{\text{second}}$

in this case, since force constant,

instantaneous power $P = \dfrac{dW}{dt} = Fv = \overline{P}$

Humans : 100 – 200 W good exercise.
400 W sustained... athlete.

746 W $\equiv$ horsepower.

In 2 or 3 dimensions...

$$dW = \vec{F}\cdot d\vec{s} \quad \text{or} \quad \vec{F}\cdot d\vec{r}$$

$d\vec{s} = d\vec{r}$ infinitesimal displacement

RHK4 KK

Think: circular orbit, gravity.

$\vec{F}\cdot d\vec{s} = 0$

Gravity does no work in this case.

Instaneous power

$$\frac{dW}{dt} = \vec{F}\cdot\frac{d\vec{s}}{dt} = \vec{F}\cdot\vec{v}$$

true when $\vec{v}$, $\vec{F}$ not constant.

pp. 140-141 RHK4 p. 186 KK

Potential Energy U associated with conservative force field ...

1-d

$$U(x) = \underbrace{\text{constant}}_{\text{arbitrary constant}} + \int \underbrace{(-F(x))}\, dx$$

F due to field

You would push with $-F$

$$\frac{dU}{dx} = -F(x) \quad \text{or} \quad F(x) = -\frac{dU}{dx}$$

Matters most is change in U

$$\Delta U = U(x) - U(x_0) = -\int_{x_0}^{x} F(x)\, dx$$

Gravity: $x \to y$ $\quad F(y) = -mg$

$$\Delta U = -\int_{y_0}^{y} (-mg)\, dy = mg\underbrace{(y - y_0)}_{h = y - y_0}$$

$$\Delta U = mgh$$

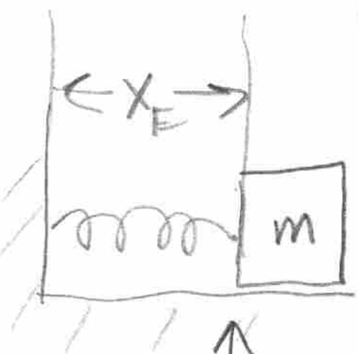
Work energy theorem . . .

$$U(x_1) + K_1 = U(x_2) + K_2$$

Gravity $mgy_1 + \frac{1}{2}mv_1^2 = mgy_2 + \frac{1}{2}mv_2^2$

Simple Harmonic Oscillator . . .

$$F = -k(x' - x_E)$$



equilibrium length

just make

$x = 0$ the equilibrium length

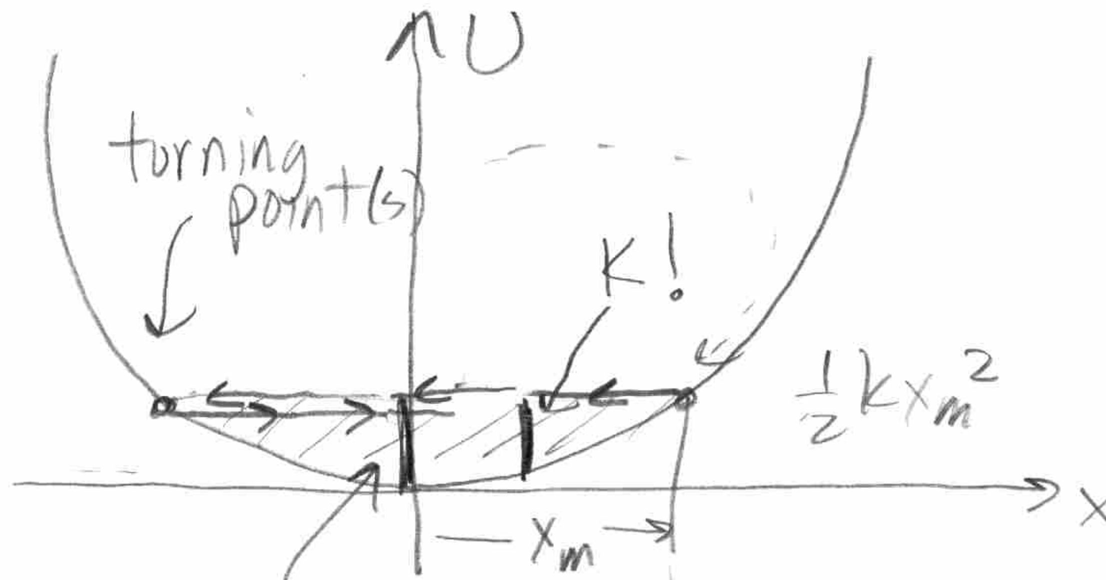
$$F = -kx$$

$$U(x) = \text{constant} + \int(--kx)\,dx$$

$$= \text{constant} + \frac{1}{2}kx^2$$

make a plot of this . . .

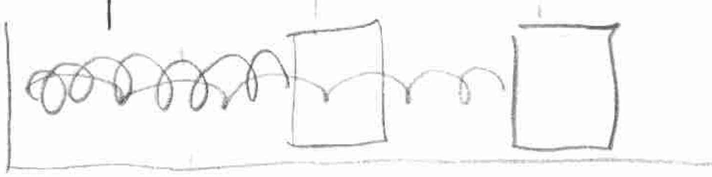
make constant $= 0$



pp 156 – 158 RHK4

pp. 176 – 182 K + K

Imagine... pulling spring to maximum x_m displacement from equilibrium



$$U(x_m) = \frac{1}{2}kx_m^2 = E = U(x) + K$$

$$\frac{1}{2}kx_m^2 = \frac{1}{2}kx^2 + K$$

$$K = \frac{1}{2}k(x_m^2 - x^2)$$

"Motion" .. oscillation between turning points ...

$$V_m \rightarrow \quad \frac{1}{2} m V_m^2 = \frac{1}{2} k x_m^2$$

$$V_m = \pm \sqrt{\frac{k}{m}}\, x_m$$

$$v_m = \pm \omega_0 x_m$$

$$T = \frac{2\pi}{\omega_0} = 2\pi \sqrt{\frac{m}{k}}.$$

More generally ... when $U(x)$ has a minimum ...

$$(N) \leftarrow x \longrightarrow (N)$$

N_2 nitrogen $\simeq 78\%$ of E

$$U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$$

x small ... $\rightarrow \dfrac{a}{x^{12}}$

x large $\rightarrow -\dfrac{b}{x^6}$