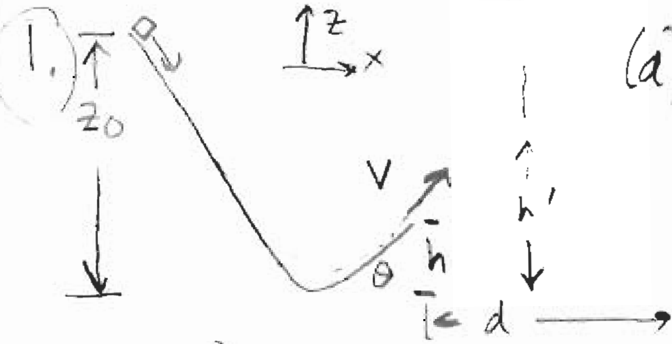
1.



(Trajectory)

(a) $\frac{1}{2}mv^2 = mg(z_0 - h)$

$v = \sqrt{2g(z_0-h)}$

vertical, horiz components

$V_{z\theta} = V\sin\theta \qquad V_{x\theta} = V\cos\theta$

vertical motion: top of trajectory is where z-component of velocity is zero happens at time t_1

$0 = V_{z\theta} - gt_1 \qquad t_1 = \frac{V_{z\theta}}{g}$

then $h' = z' + V_{z\theta}t_1 - \frac{1}{2}gt_1^2$

$= h + \frac{V_{z\theta}^2}{g} - \frac{1}{2}g\frac{V_{z\theta}^2}{g^2}$

$h' = h + \frac{V_{z\theta}^2}{2g}$ ... can also get from energy.

t_2, time to drop to ground is then

$\frac{1}{2}gt_2^2 = h' \Rightarrow t_2 = \sqrt{\frac{2h'}{g}}$

and so, $d = V_{x\theta}(t_1 + t_2)$

$d = V\cos\theta\left(\frac{V\sin\theta}{g} + \sqrt{\frac{2}{g}\left(h + \frac{V^2\sin^2\theta}{2g}\right)}\right)$

$d = \frac{V^2 \cdot \sin\theta\cos\theta}{g} + V\cos\theta\sqrt{\frac{2}{g}\left(h + \frac{V^2\sin^2\theta}{2g}\right)}$

$= \frac{2g(z_0-h)\cdot\frac{1}{2}}{g} + \sqrt{2g(z_0-h)}\cdot\frac{1}{\sqrt{2}}\sqrt{\frac{2}{g}\left(h + \frac{2g(z_0-h)\cdot\frac{1}{2}}{2g}\right)}$

$= z_0 - h + \sqrt{2(z_0-h)\left(h + \frac{1}{2}(z_0-h)\right)}$

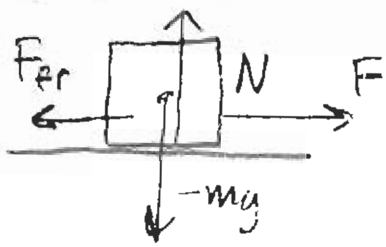
$= z_0 - h + \sqrt{2(z_0-h)\cdot\frac{1}{2}(z_0+h)}$

$$\boxed{d = z_0 - h + \sqrt{z_0^2 - h^2}}$$

(b) By inspection, d only grows smaller as h increases. You can reason this out by noting that the larger h gets, the smaller v gets, and so the smaller the horizontal component of v gets, and the shorter the time in the air becomes.

So, largest d occurs when $h = 0$, and is $d = 2z_0$

2. (a)



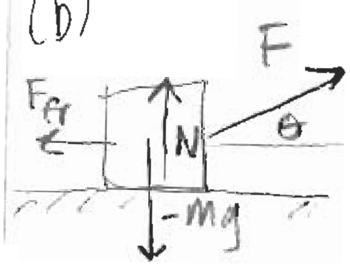
$$N - mg = 0$$
$$N = mg = 1 \cdot 10 = 10\,N$$

$$ma_x = F - F_{fr}$$

F is 4.8 N; maximum F_{fr} is $\mu \cdot N = \frac{1}{2} \cdot 10 = 5N$

Since $F < \mu N = 5N$, you cannot move horizontally

(b)



$$N + F\sin\theta - mg = 0$$

$$N = mg - F\sin\theta = 1 \cdot 10 - 4.8 \cdot \underbrace{\sin(20^\circ)}_{0.34}$$
$$N = 10 - 4.8 \cdot 0.34 = 10 - 1.64$$
$$N = 8.36\,N$$

$$ma_x = \underbrace{F\cos\theta}_{4.8 \cdot \cos(20^\circ)} - F_{fr}$$

$\cos(20^\circ) = 0.94$; $4.8 \cdot 0.94 = 4.51N$

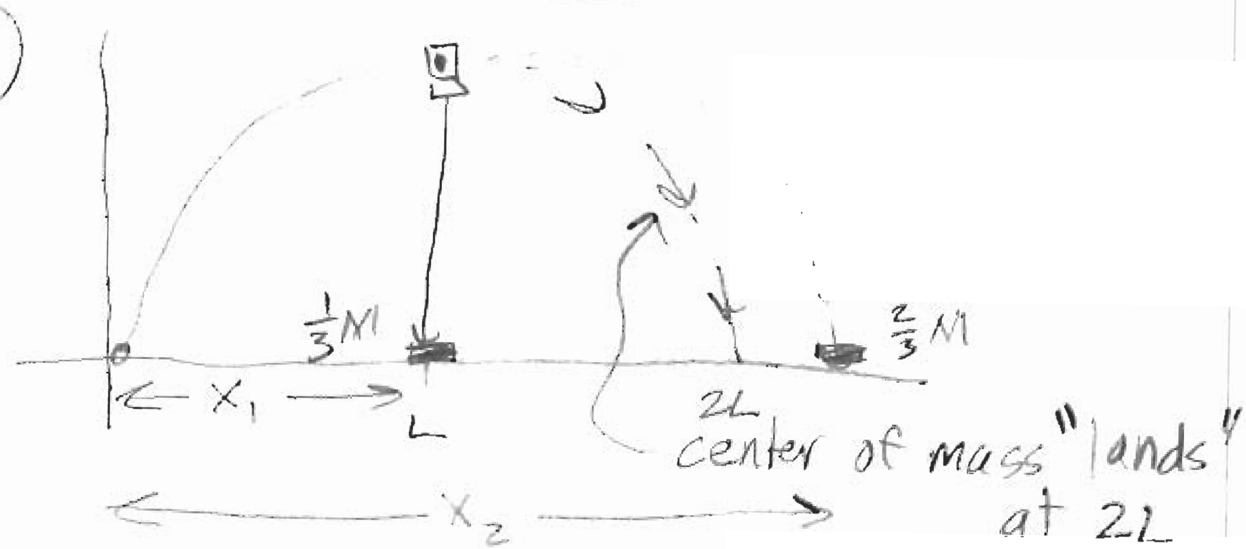
maximum F_{fr} is $\mu \cdot N = \frac{1}{2} \cdot 8.36 = 4.18\,N$

3.

$$-ma = -m\frac{V^2}{R} = -\mu m g$$

$$\boxed{\mu = \frac{V^2}{gR} = \frac{20^2}{10\cdot 20} = \frac{20}{10} = 2}$$

4.



$$\frac{\frac{1}{3}Mx_1 + \frac{2}{3}Mx_2}{M} = 2L$$

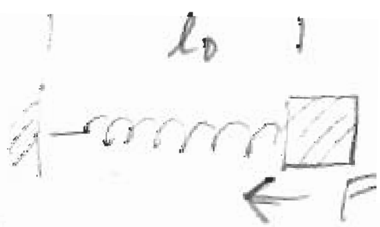
$$x_1 = L$$

$$\frac{1}{3}L + \frac{2}{3}x_2 = 2L$$

$$\frac{2}{3}x_2 = \frac{5}{3}L$$

$$\boxed{x_2 = \frac{5}{2}L}$$

5.



(a) impulse: $mv_0 = F\Delta t \Rightarrow \Delta t = \frac{mv_0}{F} = \frac{4\cdot 2}{10^3}$

$$\boxed{\Delta t = 8\cdot 10^{-3}\,s}$$

(b) $\frac{1}{2}k(\Delta x)^2 = \frac{1}{2}mv_0^2 \Rightarrow \Delta x = \sqrt{\frac{m}{k}}\,v_0 = \sqrt{\frac{4}{10^2}}\cdot 2$

[illegible]

(c) $T \quad \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{k}{m}}} = 2\pi\sqrt{\frac{m}{k}} = 2\pi\cdot\sqrt{\frac{4}{10^{-2}}}$

$T = 2\pi \cdot 2 \cdot 10^{-1} = 0.4\pi\ s$

It takes $\frac{1}{2}$ of a period to return.

$$\boxed{t = T\cdot\frac{1}{2} = 0.2\pi\ s}$$

6. $\frac{1}{2}\cancel{m}v^2 = -\frac{GM\cancel{m}}{R_e+h} - \left(\frac{-GM\cancel{m}}{R_e}\right)$

<u>note</u>: $h = \frac{R_e}{2}$ $\frac{GM}{R_e^2} = g$ so

$\frac{GM}{R_e} = gR_e$

$\frac{1}{2}v^2 = -\frac{GM}{\frac{3}{2}R_e} + \frac{GM}{R_e}$

$= \left(-\frac{2}{3}+1\right) gR_e$

$v^2 = \frac{2}{3}gR_e$

$v = \sqrt{\frac{2}{3}gR_e} = \sqrt{\frac{2}{3}\cdot 10\cdot 6.4\cdot 10^6}$

$$\boxed{v = 6530\ m/s}$$

7. (a) $U'(x) = -\frac{2A}{x^3} + \frac{B}{x^2} = 0 \leftarrow$ equilibrium

$$\frac{B}{x^2} = +\frac{2A}{x^3}$$

$$\boxed{x = 2\frac{A}{B} = 2\cdot\frac{5}{10} = 1 \text{ meter}}$$

Check: is it stable?

$$U''(x) = \frac{6A}{x^4} - \frac{2B}{x^3} = \frac{2}{x^4}(3A - Bx)$$

$$= \frac{2}{\left(2\frac{A}{B}\right)^4}\left(3A - B\cdot\frac{2A}{B}\right) = \frac{1}{8}\frac{B^4}{A^4}\cdot A^3 = \frac{1}{8}\frac{B^4}{A^3} > 0$$

YES

(b)

$$\omega = \sqrt{\frac{1}{m}U''(x)} = \sqrt{\frac{1}{m}\frac{1}{8}\frac{B^4}{A^3}}$$

$$\boxed{\omega = \frac{1}{2}\sqrt{\frac{B^4}{2mA^3}}}$$ symbolically

numerically $\omega = \frac{1}{2}\cdot\sqrt{\frac{1}{8}\frac{1}{\left(\frac{1}{5}\right)}\cdot\left(\frac{10}{5}\right)^3\cdot 5}$

$$= \frac{1}{2}\sqrt{\frac{1}{8}\cdot 5\cdot 2^3\cdot 5}$$

$$\boxed{\omega = 5/2 \text{ rad/s}}$$

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8. (a)

Initial

Intermediate: $V = \frac{m_1}{m_1+m_2} u_1$

C.M.: $u_1 - V = u_1 - \frac{m_1}{m_1+m_2} u_1 = \frac{m_2}{m_1+m_2} u_1$

after scatter.. m_1 has speed $-\frac{m_2}{m_1+m_2} u_1$
in center of mass frame

in original frame $v_1 = -\frac{m_2}{m_1+m_2} u_1 + V$

$$= -\frac{m_2}{m_1+m_2} u_1 + \frac{m_1}{m_1+m_2} u_1$$

$$v_1 = \frac{m_1 - m_2}{m_1+m_2} u_1 = f u_1$$

$$\frac{m_1 - m_2}{m_1+m_2} = f$$

$$m_1 - m_2 = f m_1 + f m_2$$

$$(1-f) m_1 = (1+f) m_2$$

$$\boxed{m_2 = \frac{1-f}{1+f} m_1}$$

(b) $\boxed{m_2 = \frac{1-(-\frac{1}{2})}{1-\frac{1}{2}} = \frac{3}{2} m_1 = 1.5 \text{ kg}}$