# Physics 21 Problem Set 8

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**due Monday, Feb. 28 at 5pm**

**Course Announcements:**
Reading for these Problems: RHK4 14-4 and 14-5, 15-1 through 15-6, KK 10.1.
PSR Fellows, who are advanced Physics Majors, are available to help you in the PSR Wed. & Thurs. from 6-8pm, and Sunday in 1640 Broida, 6-8pm.

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1. (RHK4 14.40) A cube of uniform density, mass $m$, and edge length $a$ is balanced on a cylindrical surface of radius $r$ as shown in Fig. 1. Show that the criterion for stable equilibrium of the cube, assuming that friction is sufficient to prevent slipping, is $r > a/2$. The key to this problem is considering whether the center of mass of the cube increases or decreases in elevation when the cube tilts and moves. A clever diagram gives the answer away, as well as understanding the meaning of the tilt angle $\theta$ being in radians.

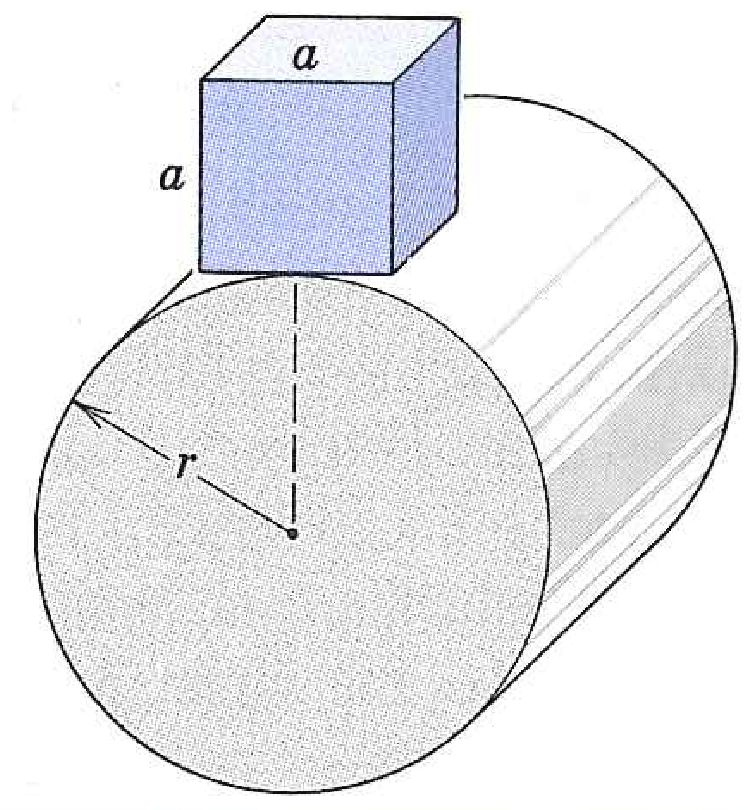


Figure 1: Problem 1.

2. (RHK4 14.46) A uniform bar of mass $m = 4.7\,\mathrm{kg}$ and length $L = 1.3\,\mathrm{m}$ is suspended at the ends by two vertical wires. Both wires are cylindrical, and one is steel with a diameter of $d_1 = 1.2\,\mathrm{mm}$, while the other is aluminum and has a diameter of $d_2 = 0.84\,\mathrm{mm}$. Before the bar was attached, the wires were of the same 1.7 m length. Find the angle $\theta$ between the bar and the horizontal; see Fig 2. You can assume that $\theta$ is so small that $\sin\theta = \theta$, as long as $\theta$ is in radians. Ignore the change in diameters of the wires; the bar and wires are in the same plane. A table of Young's moduli and other quantities is in Fig. 3.

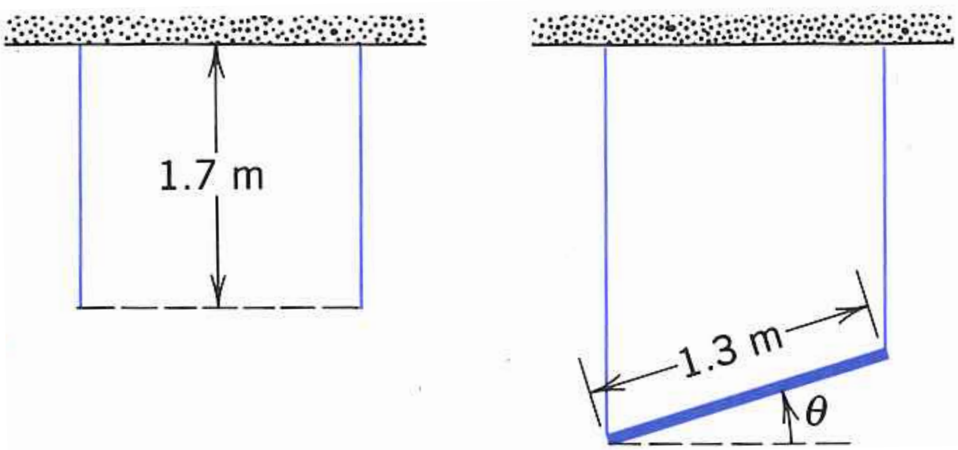


Figure 2: Problem 2.

**TABLE 1 SOME ELASTIC PROPERTIES OF SELECTED MATERIALS OF ENGINEERING INTEREST**

| *Material* | *Density* (kg/m³) | *Young's Modulus* ($10^9$ N/m²) | *Ultimate Strength* ($10^6$ N/m²) | *Yield Strength* ($10^6$ N/m²) |
|---|---|---|---|---|
| Steel[a] | 7860 | 200 | 400 | 250 |
| Aluminum | 2710 | 70 | 110 | 95 |
| Glass | 2190 | 65 | 50[b] | — |
| Concrete[c] | 2320 | 30 | 40[b] | — |
| Wood[d] | 525 | 13 | 50[b] | — |
| Bone | 1900 | 9[b] | 170[b] | — |
| Polystyrene | 1050 | 3 | 48 | — |

[a] Structural steel (ASTM-A36).
[b] In compression.
[c] High strength.
[d] Douglas fir.

Figure 3: Problem 2.

3. (RHK4 15.8) A body oscillates with simple harmonic motion according to the equation:

$$x(t) = (6.12\,\mathrm{m})\cos[(8.38\,\mathrm{rad/s})t + 1.92\,\mathrm{rad}].$$

Find, numerically,

(a) the displacement as a function of time, and at the specific time $t = 1.90\,\mathrm{s}$;

(b) the velocity as a function of time, and at the specific time $t = 1.90\,\mathrm{s}$;

(c) the acceleration as a function of time, and at the specific time $t = 1.90\,\mathrm{s}$;

(d) the frequency (*not* circular frequency); and

(e) the period of the motion.

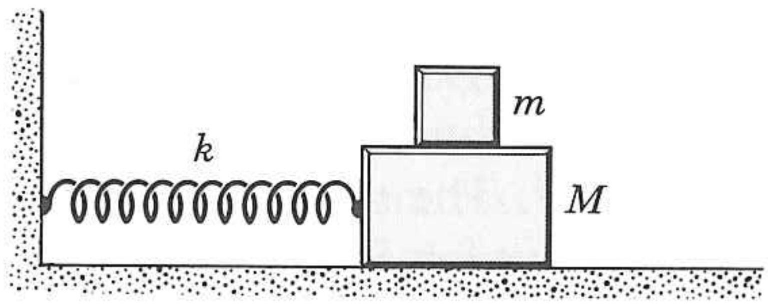


Figure 4: Problem 4.

4. (RHK4 15.14) Two blocks ($m = 1.22\,\mathrm{kg}$ and $M = 8.73\,\mathrm{kg}$) and a spring ($k = 344\,\mathrm{N/m}$) are arranged on a horizontal, frictionless surface as shown in Fig. 4. The coefficient of static friction between the blocks is $\mu_s = 0.42$. Find the maximum possible amplitude of the simple harmonic motion if no slippage is to occur between the blocks.

5. (RHK4 15.30) A 5.13 kg object moves on a horizontal frictionless surface under the influence of spring with force constant $k = 9.88\,\mathrm{N/cm}$. The object is displaced from equilibrium $x(0) = 53.5\,\mathrm{cm}$ and given an initial velocity of $v(0) = 11.2\,\mathrm{m/s}$ gack toward the equilibrium position. Find

   (a) the frequency of the motion,

   (b) the initial potential energy $U(0)$ of the system,

   (c) the initial kinetic energy $K(0)$ of the system,

   (d) the amplitude of the motion, and

   (e) make a plot of energy versus displacement from equilbrium, showing $U(x)$, $U(0)$, $K(0)$, the total energy, and the turning points.

6. Make a table with 6 columns, where the first column is the angle $\theta$ in radians, and make the 12 entries in the first column $\theta = 0.001, 0.002, 0.005, 0.01, 0.02, 0.05, 0.1, 0.2, 0.5, 0.75, 1, 1.1$ radians. Then make the second column $\sin\theta$, and evaluate it for 12 $\theta$ values given. Make the third column the percentage difference between $\theta$ and $\sin\theta$, that is, $100 \times (\theta - \sin\theta)/\sin\theta$. The last three columns make $(1 - \cos\theta)$, $\theta^2/2$, and percentage difference between $\theta^2/2$ and $1 - \cos\theta$. In the each case, at what given value of $\theta$ is the percentage difference closest to 10%?

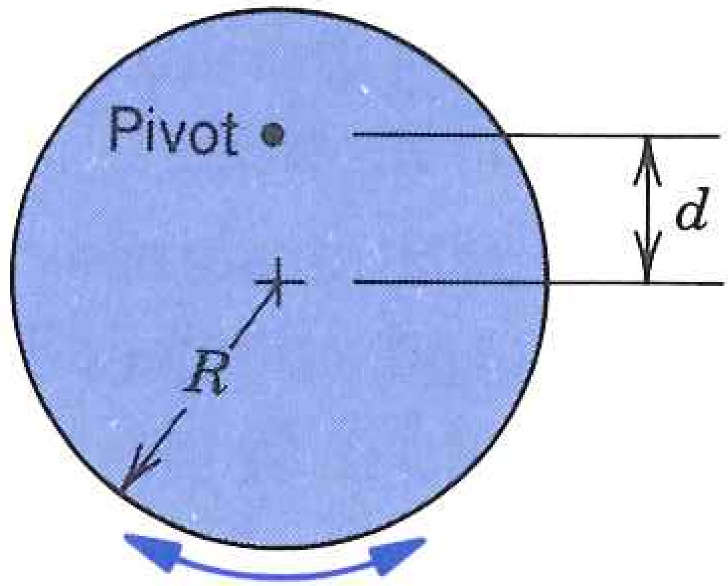


Figure 5: Problem 7.

7. (RHK4 15.45) A physical pendulum consists of a uniform solid disk of mass $M = 563\,\text{g}$ and a radius $R = 14.4\,\text{cm}$ supported in a vertical plane by a pivot located a distance $d = 10.2\,\text{cm}$ from the center of the disk, as shown in Fig. 5. The disk is displaced by a small angle and released. Find the period of the resulting simple harmonic motion.

8. (RHK4 15.49) A pendulum is formed by pivoting a long thin uniform rod of length $L$ and mass $m$ about a point on the rod which is a distance $d$ above the center of the rod.

   (a) Evaluate the small-amplitude period of this pendulum in terms of $d$, $L$, $m$, and $g$.

   (b) Show that the period has a *minimum* value when $d = L/\sqrt{12} \approx 0.289L$.

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