Center of Mass Coordinates

Often a problem can be simplified by the right choice of coordinates. The center of mass coordinate system, in which the origin lies at the center of mass, is particularly useful. The drawing illustrates the case of a two particle system with masses $m_1$ and $m_2$. In the initial coordinate system, $x$, $y$, $z$, the particles are located at $r_1$ and $r_2$ and their center of mass is at

$$R = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2}.$$

We now set up the center of mass coordinate system, $x'$, $y'$, $z'$, with its origin at the center of mass. The origins of the old and new system are displaced by $R$. The center of mass coordinates of the two particles are

$$r'_1 = r_1 - R,$$

$$r'_2 = r_2 - R.$$

Center of mass coordinates are the natural coordinates for an isolated two body system. For such a system the motion of the center of mass is trivial—it moves uniformly. Furthermore,
Largest known trans-Neptunian objects (TNOs)

- Dysnomia
- Nix
- Charon
- Hydra
- Namaka
- Hi'iaka
- Eris
- Pluto
- Makemake
- Haumea
- Sedna
- Orcus
- Vanth
- Weywot
- 2007 OR₁₀
- Quaoar
Fig. 1. (90) Antiope system observed with VLT NACO in 2004. The two components of the doublet system are clearly identified on these basic-processed (sky subtraction, flat-fielding, bad pixel removal) near infrared observations. The relative positions of the two components can be found in Table 1. We also displayed on the far right three PSF frames. Because the FWHM of the PSF is similar to the FWHM on the individual components of the double system, we can deduce that the two components cannot be resolved individually by the AO system. Their angular size is below the diffraction limit of the telescope (60 milli-arcsec for the VLT). The July 2004 observation was taken under very poor seeing conditions. In this case, the binary nature of (90) Antiope cannot be revealed.

Fig. 2. Top row is 2D images and bottom row is a 3D representation of brightness distribution recorded using the Keck AO system on May 31, 2005. Left column is the raw profile. A residual halo due to the imperfection of the AO system can be seen. Middle column is the bi-dimensional fitted Gauss–Moffat profile. Right column shows the residuals of the fit. Which is quite accurate with low residuals (less than 2% the peak value). On this image the two components of the system are separated by 0.125 arcsec.
Earth-Moon System rotates about its center of mass.

**Binary Asteroids**: (recent)

**Coefficient of Mass**: \( m_1, m_2 \)

**Mass Distribution**: \( M = \frac{m_1 m_2}{m_1 + m_2} \)

**Center of Mass**: \( \mu = \frac{1}{m_1} + \frac{1}{m_2} \)

**Position Vector**: \( \vec{r} = \vec{r}_1 - \vec{r}_2 \)

**Force Vector**: \( \vec{F} = \frac{\vec{r}}{|\vec{r}|^2} \)

**Interaction Force**: \( \vec{F}_{int,1} = -G \frac{m_1 m_2}{r^2} \)

**Gravity Force**: \( \vec{F}_{int,1} = -G \frac{m_1 m_2}{r^2} \)
\[
\frac{m_1 m_2}{m_1 + m_2} \frac{d^2 r}{dt^2} = - \frac{G (m_1 + m_2)}{r^2} r
\]

Assume circular:
\[
\frac{d^2 r}{dt^2} = - \frac{G (m_1 + m_2)}{r^2} r
\]

\[
V = \left[ \frac{G (m_1 + m_2)}{r} \right]^{1/2}
\]

\[
T = \frac{2\pi r}{V} = \left[ \frac{2\pi r}{G (m_1 + m_2)} \right]^{1/2}
\]

\[
T^2 = \frac{(2\pi)^2 r^3}{G (m_1 + m_2)}
\]

\[
(m_1 + m_2) = \frac{(2\pi)^2 r^3}{G T^2}
\]

90 Antiope: show figure

\[
r = 171 \text{ km} \quad T = 16.5 \text{ hours}
\]

\[
(m_1 + m_2) = \frac{(2\pi)^2 \cdot (171 \cdot 10^3)^3 \text{ m}^3}{2 \cdot 10^{-10} \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2}} \cdot (16.5 \cdot 3600)^2
\]

\[
(m_1 + m_2) \approx 8.4 \cdot 10^{17} \text{ kg}
\]

Density \( \approx 1.25 \text{ g/cm}^3 \)
→ no CM continuous distribution.

→

\[
\begin{align*}
\text{\#2} & \quad \text{\#1} \\
M \times m & \quad a + \text{rest} \\
\vec{p}_{\text{initial}} & = 0
\end{align*}
\]

\[
\begin{align*}
\vec{p}_2 & = MV \uparrow \\
\end{align*}
\]

→ only internal forces

\[
\begin{align*}
\frac{d}{dt}(\vec{p}_1 + \vec{p}_2) & = 0 \\
\vec{p} & = \vec{p}_1 + \vec{p}_2 = \vec{0} \\
-MV + mv & = 0 \\
\frac{v}{\sqrt{V}} & = \frac{M}{m}
\end{align*}
\]

use in your problem

what did happen. an impulse

\[
\begin{align*}
\vec{r} & = \vec{r}_1 - \vec{r}_2 \\
\end{align*}
\]

\[
\begin{align*}
\vec{v}_0 & = \vec{F}_{\text{int}} \\
\end{align*}
\]
\[
\dot{r}(0) = 0, \quad \mu r(t) - r(0) = \int F_{\text{int}} \, dt = \vec{P}_f - \vec{P}_i.
\]

the impulse \( \vec{I} \) (my symbol)

Two types here:

1. \( F_{\text{int}} \) from an explosion
   - large, brief

\[
\begin{array}{c|c}
F_{\text{int}} & \text{AREA} \\
\hline
\text{matters} & (\text{shape... no big deal})
\end{array}
\]

\( \mu \rightarrow m \)

then \( \vec{P}_f - \vec{P}_i = \vec{I} \)

\( M_* \rightarrow \infty \)

\( m = 0.2 \text{m} \)

\( V_0 = 8 \text{ m/s} \)

\( \vec{P}_i = -m V_0 \hat{\jmath} \)

\( \vec{P}_f = m V_0 \hat{\jmath} \)
Impulse then \( m\mathbf{v}_0 \hat{j} - (-m\mathbf{v}_0 \hat{j}) = \mathbf{I} \)

\[
\mathbf{I} = -2m\mathbf{v}_0 \hat{j} = -2 \times 0.2 \times 0.8 \mathbf{j} = 3.2 \text{ } \frac{\text{kg-m}}{s}
\]

\[
\Delta t = 10^{-3} \text{ s}
\]

\[
\mathbf{F}_{\text{avg}} \cdot \Delta t = 3.2 \mathbf{j} \quad \text{kg-m/s}
\]

\[
\mathbf{F}_{\text{avg}} = \frac{3.2}{10^{-3}} \mathbf{j} \quad \text{kg-m/s}^2
\]

\[
\mathbf{F}_{\text{avg}} = 3200 \mathbf{j} \text{ N}
\]

Balls' acceleration \( \mathbf{F}_{\text{avg}} \) \( \frac{\mathbf{F}_{\text{avg}}}{m} \approx 16,000 \text{ m/s}^2 \)

Neglected gravity,

2. Truly constant force

\[
\mathbf{F}_{\text{grav}} \cdot \Delta t = 0.2 \times 9.8 \times 10^{-3} \text{ N}
\]

\[
\approx 2 \times 10^{-3} \text{ N}
\]

Negligible
To avoid broken limbs, make $\Delta t$ as long as possible when you fall:

- **Bad**
- **Good**

$F_{\text{arg}} \sim \frac{2Mv}{\Delta t}$

Continuous tiny impulses = force

- Water
- Each
- In time $T$, $\frac{v_0}{e}$ drops hit.
- Total momentum transfer:
  $$= \left( \frac{v_0 T}{e} \right), \left( mv_0 \right)$$