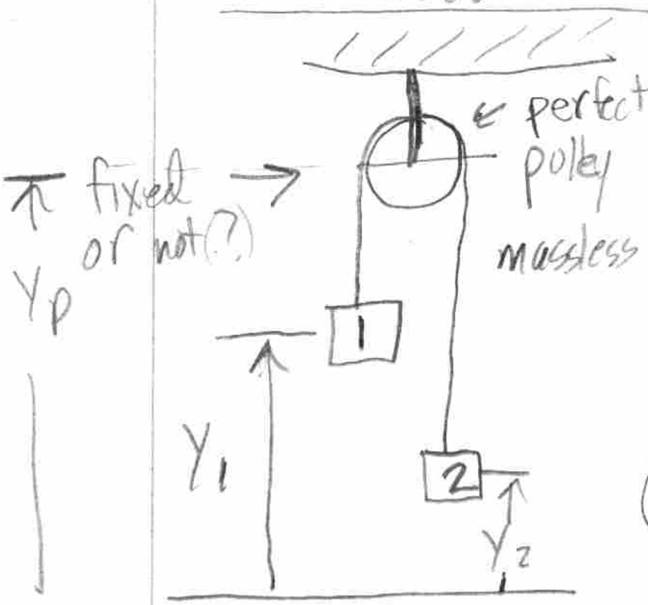
Atwood's Machine



can slow down gravity!

Qualitatively,

(1) $m_2 > m_1$ falls on right side ($m_2 < m_1$, left?)

(2) What arrangement of masses gives the largest tension in cord? one massive, one light, non zero.

Fixed polley: $\ddot{y}_1 = -\ddot{y}_2$ (intuitive)

Moving polley (!) R polley

$$(y_p - y_1) + (y_p - y_2) + \pi R = L$$

$$2\ddot{y}_p - \ddot{y}_1 - \ddot{y}_2 = 0$$

or $\ddot{y}_p = \frac{1}{2}(\ddot{y}_1 + \ddot{y}_2) = \left[= 0, \text{ fixed then } \ddot{y}_1 = -\ddot{y}_2 \right]$

$$T - m_1 g = m_1 \ddot{y}_1$$

$$T - m_2 g = m_2 \ddot{y}_2$$

$$(-m_1 + m_2) g = m_1 \ddot{y}_1 - m_2 \ddot{y}_2 = (m_1 + m_2) \ddot{y}_1$$

$$\ddot{y}_1 = \underbrace{\frac{(m_2 - m_1)}{(m_1 + m_2)}}_{\frac{\text{Mass}}{\text{Mass}}} g \qquad \begin{array}{l} >0,\ m_2 > m_1 \\ <0\ \ m_1 < m_2 \end{array}$$

acceleration $\left(\ell / T^2 \right)$

$$\ddot{y}_2 = -\ddot{y}_1 = \frac{(m_1 - m_2)}{(m_1 + m_2)} g$$

SLOWING by $m_1 - m_2$

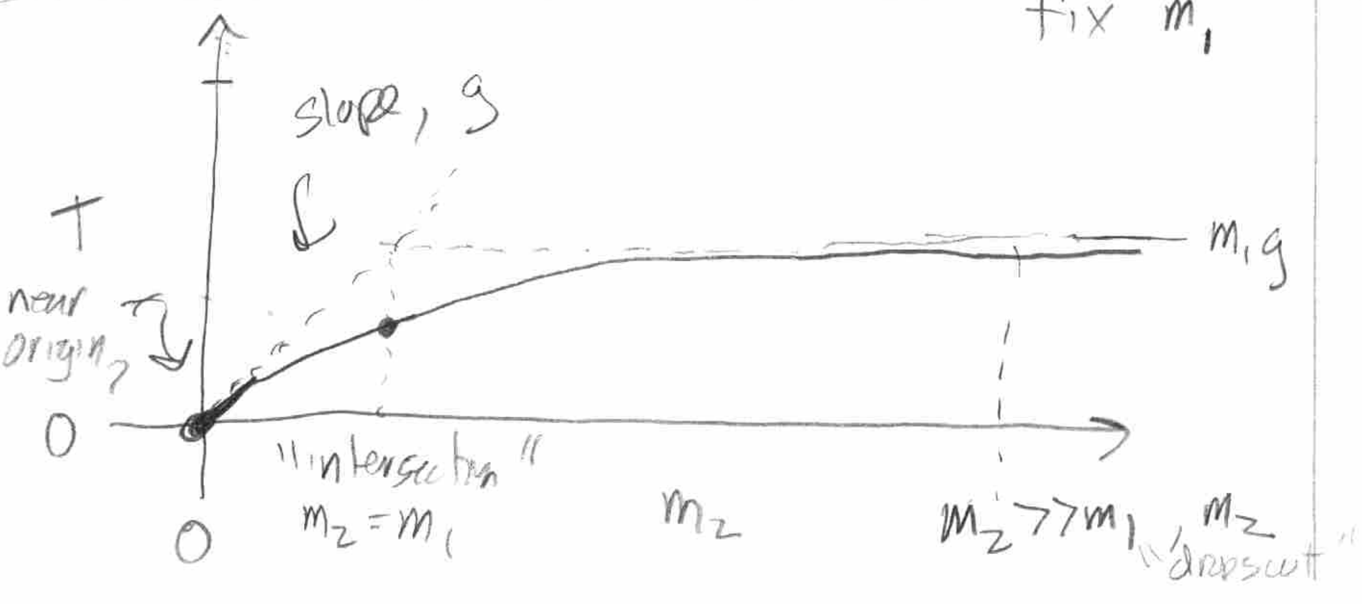
$$T = m_1 \ddot{y}_1 + m_1 g = m_1 (\ddot{y}_1 + g)$$

$$= m_1 \left(\frac{m_2 - m_1}{m_1 + m_2} + 1 \right) g$$

$$= m_1 \left(\frac{m_2 - \cancel{m_1} + \cancel{m_1} + m_2}{m_1 + m_2} \right) g$$

$$T = \frac{m_1 m_2}{m_1 + m_2} g = \mu g$$

fix m_1



$$\frac{dT}{dm_2} = \left(\frac{m_1}{m_1+m_2} - \frac{m_1 m_2}{(m_1+m_2)^2}\right) g$$

$$= \frac{m_1(m_1+m_2) - m_1 m_2}{(m_1+m_2)^2}\, g$$

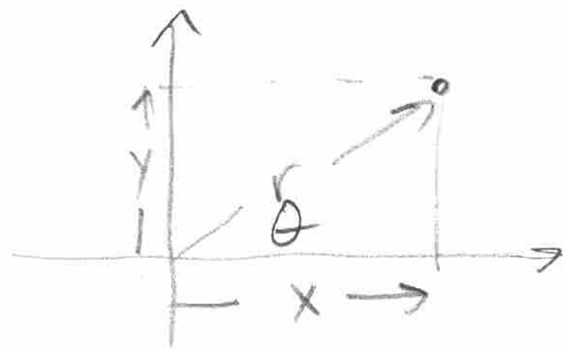
$$\frac{dT}{dm_2} = \frac{m_1^2}{(m_1+m_2)^2}\, g$$

$$\left.\frac{dT}{dm_2}\right|_{m_2=0} = g \qquad \left.\frac{dT}{dm_2}\right|_{m_2\to\infty} = 0$$

$m_2 = m_1$

$$T = \frac{m_1^2}{m_1+m_2}\cdot g = \frac{1}{2} m_1 g$$

Polar Coordinates (p27, 1.9)



$$r = \sqrt{x^2+y^2}$$

$$\tan\theta = \frac{y}{x}$$

$$\theta = \underbrace{\tan^{-1}\left(\frac{y}{x}\right)}_{\text{sign of } x \,\&\, y \text{ individually}}$$

Unit Vectors

Rectilinear: $\hat{\imath}$: increase in x

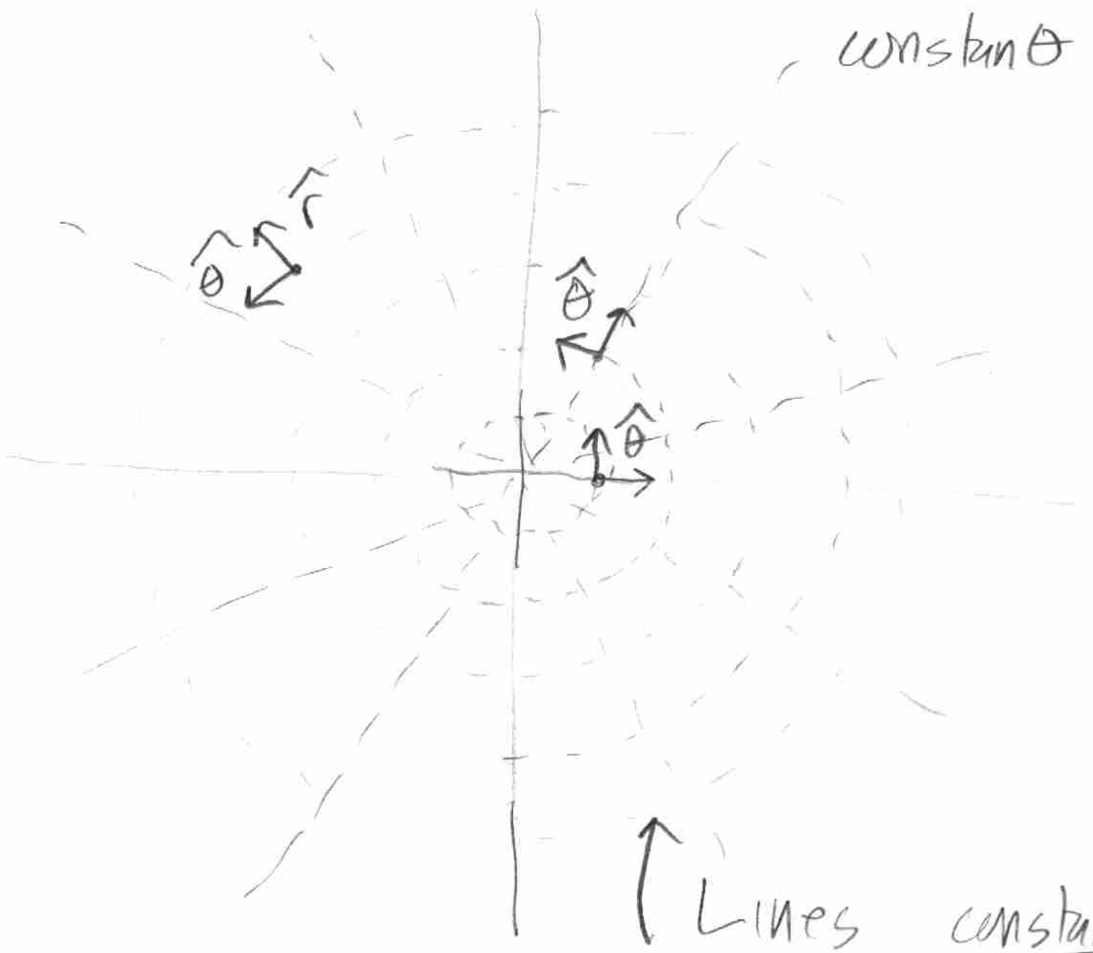
$\hat{\jmath}$ increase in y

same direction whatever x and y are

← direction of $\hat{\imath}$ & $\hat{\jmath}$ independent of x, y.

NOT TRUE for $\hat{r}$ .. direction of increasing r

$\hat{\theta}$ .. direction of increasing θ



↑ Lines constant r

$\hat{r}$ is $\perp$

$|\hat{\theta}| = |\hat{r}| = 1$

$\hat{r} \cdot \hat{\theta} = 0$ always

but they point different directions depending on where you are

$\hat{r}$ ... same direction as $x\hat{i} + y\hat{j}$