**Constant Velocity**

\[ x = A + B \frac{t}{t} \]

\[ [x] = l \]

position when \( t = 0 \), aka \( x_0 \)

\[ [A] = l \text{ (length)} \quad B \text{ aka } V \]

\[ x = x_0 + V \uparrow \]

\[ \bar{x} = x_0 + \frac{1}{2} V(t_1 + t_2) \]

**Constant Acceleration**

\[ V = B + C t = v_0 + a t \]

\[ [C] = \frac{L}{t^2} \]

\[ \bar{V} = v_0 + \frac{1}{2} a (t_1 + t_2) \]

\[ \bar{V} = v_0 + \frac{1}{2} a t \]

Note: when \( t_1 = 0 \), \( t_2 = t \)
Want $x(t)$.

**Method #1, "algebraic"**

What is the form of this curve?

\[ x = x_0 + v t \]

\[ = x_0 + (v_0 + \frac{1}{2}at)t \]

\[ x = x_0 + v_0 t + \frac{1}{2}at^2 \]

\[ v = \frac{dx}{dt} = v_0 + 2 \cdot \frac{1}{2}at = v_0 + at \]

\[ a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = a \]

**Method #2, "geometric"**

Displacement is area under the velocity curve.
(a) "go with it"

When constant acceleration

\[ \text{Area} = \frac{1}{2} (v_0 + v_0 + at) \times t = v_0 t + \frac{1}{2} at^2 \]

**Position** = (initial position) + displacement

\[ x = x_0 + v_0 t + \frac{1}{2} at^2 \]

(b) more detailed

\[ \sum v(t) \Delta t = \text{Area} \]

\[ \int v(t) \, dt = \text{displacement} \]
\[ \text{homework}: \quad x(t) = A + Bt + Ct^2 + Dt^3 \]
\[ v(t) = \dot{x}(t) = B + 2Ct + 3Dt^2 \]
\[ a(t) = \ddot{x}(t) = 2C + 6Dt \]

When \( a(t) \) is constant \( (D = 0) \),
can combine \( v = v_0 + at \)
\[ x = x_0 + v_0 t + \frac{1}{2} at^2 \]

to eliminate \( t \). What good is your last problem? PS#2

\[ t = \frac{v - v_0}{a} \]
\[ (x - x_0) = v_0 \left( \frac{v - v_0}{a} \right) + \frac{1}{2} a \left( \frac{v - v_0}{a} \right)^2 \]

Displacement \[ \approx \left( \frac{v - v_0}{a} \right) \left( v_0 + \frac{1}{2} (v - v_0) \right) \]
\[ (x - x_0) = \frac{1}{2} \frac{1}{a} (v - v_0) (v + v_0) \]

\[ v^2 - v_0^2 = 2a(x - x_0) \] \{ "potential energy" \}
Gravity near earth's surface makes a uniform downward acceleration of $a = -g$

$g = 9.8 \text{ m/s}^2$ (10 m/s$^2$ often OK)

Measure velocity with $h$

launch a ball with $v_0$ upward

suppose it rises to height $y_{max}$ (above release position!?) what was initial velocity

$y = y_{max}$
\[ y^2 - v_0^2 = 2(-g)(v_{\text{max}} - 0) \]

\[ v_0 = \sqrt{2g v_{\text{max}}} \]

\[ v_{\text{max}} = 5 \text{ m} \]

\[ v_0 \approx \sqrt{2 \times 10 \frac{\text{m}}{\text{s}^2} \times 5 \text{ m}} = \sqrt{100} \frac{\text{m}^2}{\text{s}^2} \]

\[ v_0 \approx 10 \frac{\text{m}}{\text{s}} \]

\[ 1 \frac{\text{m}}{\text{s}} = 2.24 \text{ mph} \]

\[ v_0 \approx 22 \text{ mph} \]

Suppose \[ v_0 = 100 \text{ mph} \] \( (\approx 5 \times 22) \)

\[ v_{\text{max}} \propto v_0^2 \]

\[ v_{\text{max}} \approx 25 \times 5 \text{ m} \]

\[ \approx 125 \text{ meters} \]

\[ \uparrow \]

\[ 4 \text{ meters/story} \]

\[ \approx 30 \text{ story building} \]

\[ v = v_0 - g t \]

\[ 0 = v_0 - g t_{\text{max}} \]

\[ t_{\text{max}} = \frac{v_0}{g} = \frac{10 \frac{\text{m}}{\text{s}}}{10 \frac{\text{m}}{\text{s}^2}} = 1 \text{ s} \]
How long to fall back?

another is \( v = v_0 - t \).