Displacement: free vector

Position: bound vector

Displacement: start (x_1, y_1, z_1)

end (x_2, y_2, z_2)

$$\vec{S} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

NOTE: α goes from $(0,0,0)$ m

to $(1,2,3)$ m

$$\vec{S}_\alpha = (\hat{i} + 2\hat{j} + 3\hat{k})\ \text{m}$$

β goes from $(-3, 8, 17)$

to $(-2, 10, 20)$

$$\vec{S}_\beta = (\hat{i} + 2\hat{j} + 3\hat{k})$$

$\vec{S}_\alpha = \vec{S}_\beta$] starting point not important for displacement

The Position Vector

$$x\hat{i} + y\hat{j} + z\hat{k}$$

α: $\left.\begin{aligned}\vec{r}_{1\alpha} &= 0 \\ \vec{r}_{2\alpha} &= 1\hat{i} + 2\hat{j} + 3\hat{k}\end{aligned}\right\}$ $\vec{S}_\alpha = \vec{r}_{2\alpha} - \vec{r}_{1\alpha}$

$$\beta: \quad \left.\begin{aligned} \vec{r}_{1\beta} &= -3\hat{i} + 8\hat{j} + 17\hat{k} \\ \vec{r}_{2\beta} &= -2\hat{i} + 10\hat{j} + 20\hat{k} \end{aligned}\right\} \quad \vec{s}_\beta = \vec{r}_{2\beta} - \vec{r}_{1\beta}$$

- Position vectors depend on <u>where</u> you put the origin of your coordinate system ("Bound" vector).
- Displacement vectors <u>do</u> <u>not</u>] a bit more fundamental

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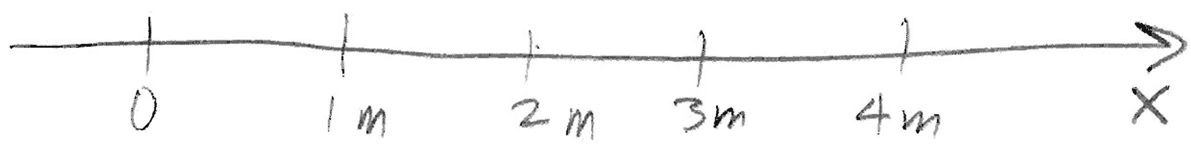
Motion in one dimension (1.6)

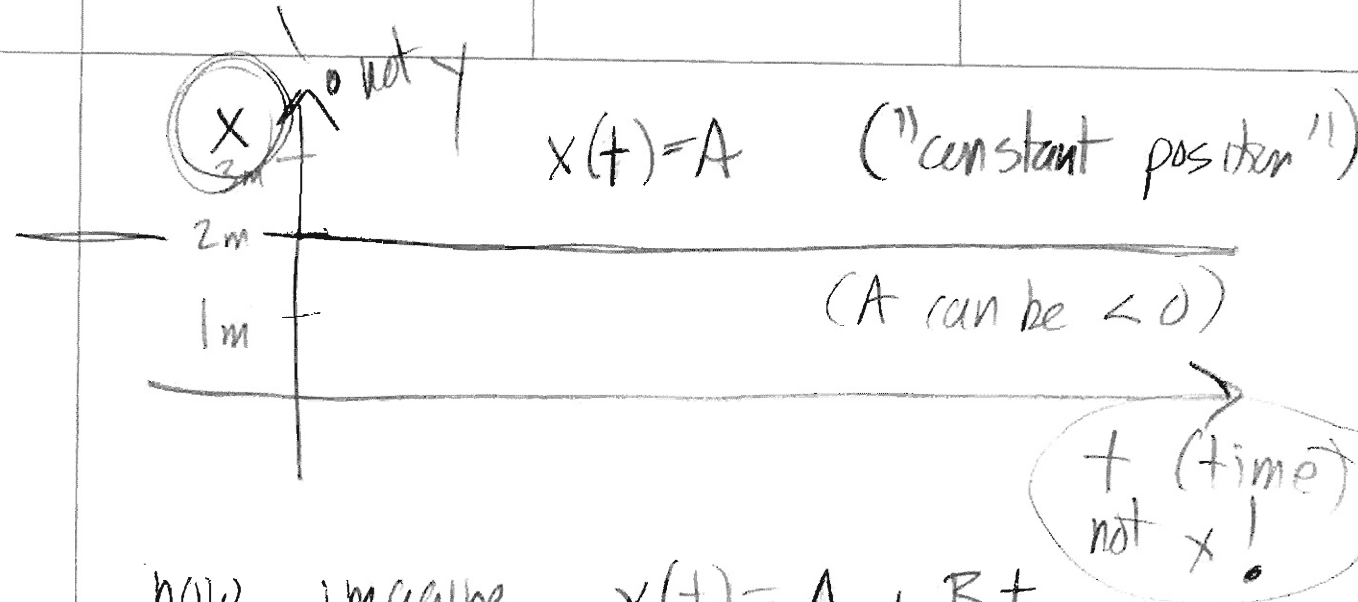


→ "point" particle
→ if massive, applies to its center of mass
→ suppose it sits, at, say, $x = 2m$ <u>FOREVER</u> (all time)

$$x(t) = 2m = A \qquad (A = 2m)$$

plot that...

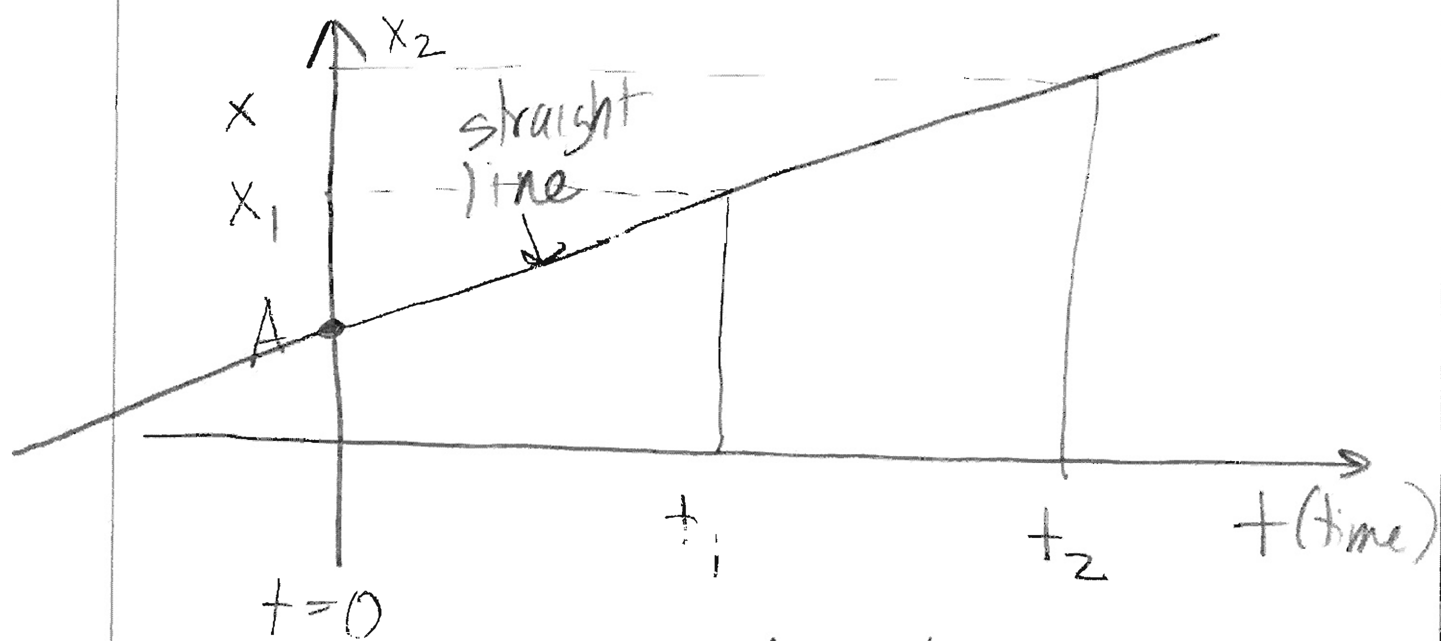
$x(t) = A$ ("constant position")



now imagine $x(t) = A + Bt$

("constant velocity")

graph:



$$x_1 = A + Bt_1$$
$$x_2 = A + Bt_2$$

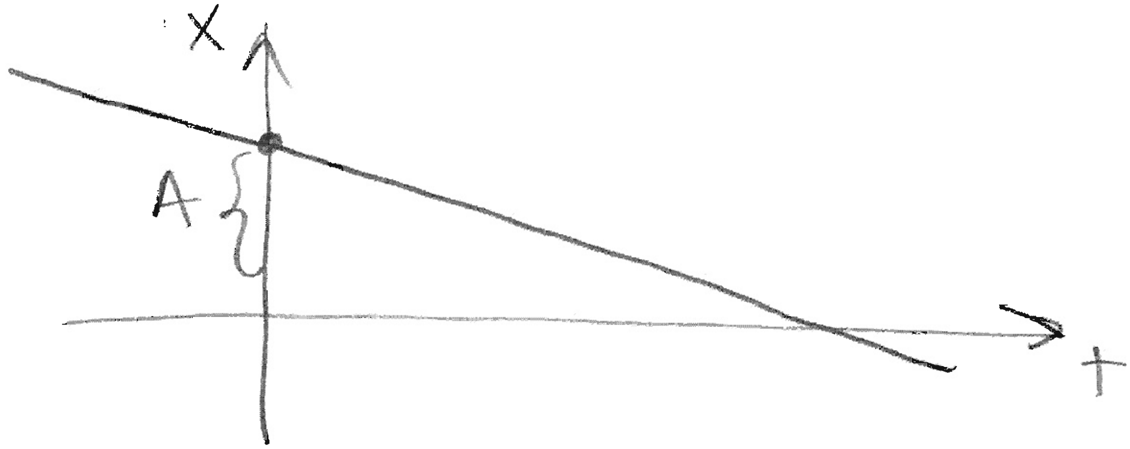
slope (rise over run)

$$= \frac{x_2 - x_1}{t_2 - t_1} = \frac{\text{displacement}}{\text{time interval}} = \frac{\cancel{A} + Bt_2 - \cancel{A} - Bt_1}{t_2 - t_1}$$

$$= B\frac{t_2 - t_1}{t_2 - t_1} = B \quad \text{independent of } t_2 - t_1.$$

Imagine $t_2 \to t_1$ ... get B for slope no matter how close t_2 gets to t_1.

B: can be < 0
can be > 0



B is a 1-d vector
called velocity.

|B| called speed

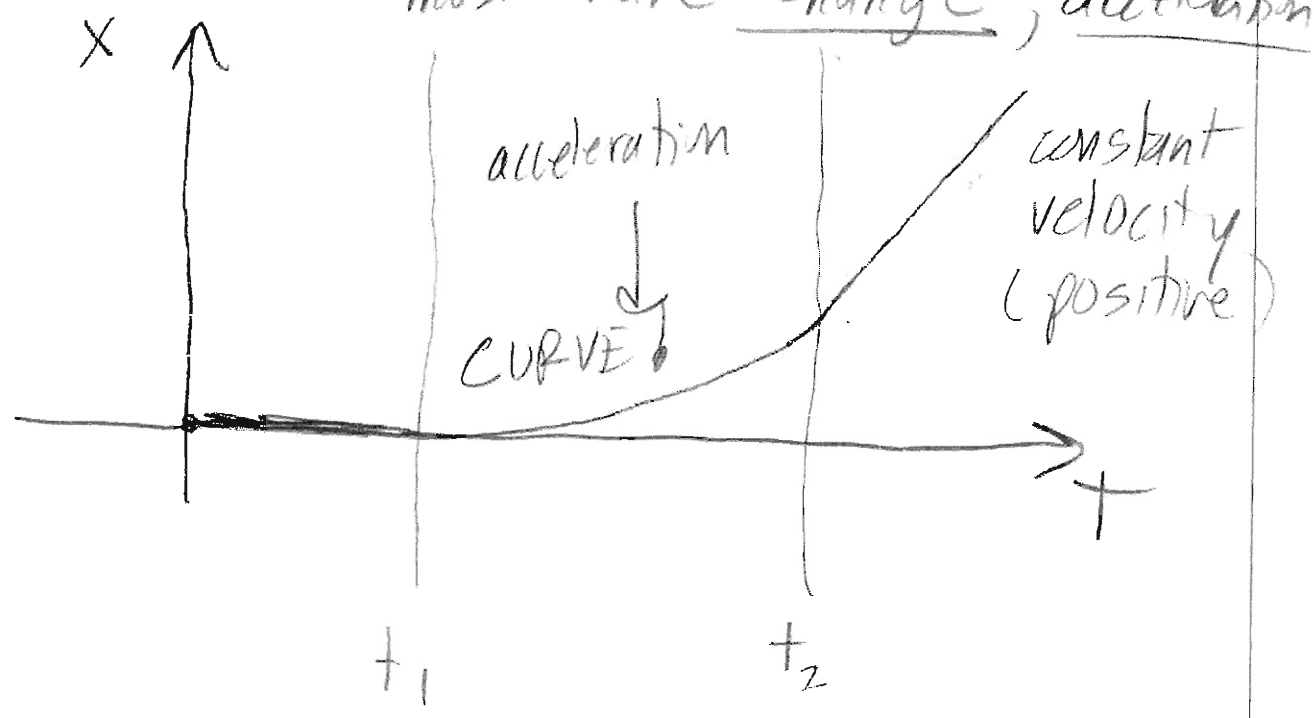
more fun in 2-d.

constant speed $\neq$ constant velocity
give example

Einstein: If you were moving with constant velocity, without external influence (like, air...) you see all the same laws of physics as someone else moving with a different constant velocity. Relativity.

Acceleration = change in velocity is where interesting stuff happens particularly in > 1-d (1-d for now)

Best example: start from rest ($x=0$, say) get to constant velocity must have change, acceleration



VISUALLY:

acceleration (holds water)

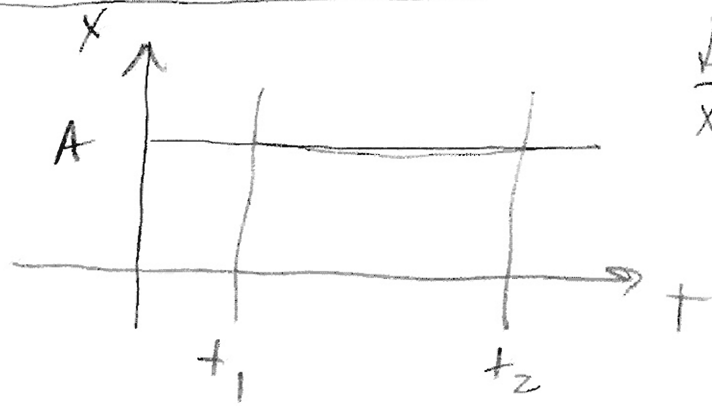
acceleration (dumps water).

Average, Instantaneous

Position
Velocity
Acceleration

(1-d vectors all)

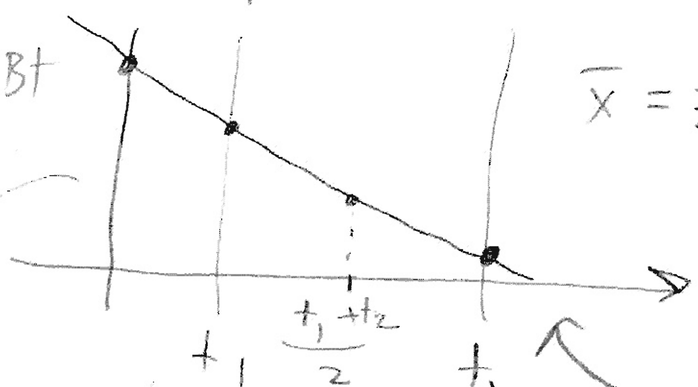
in a time interval → t_1 to t_2



average

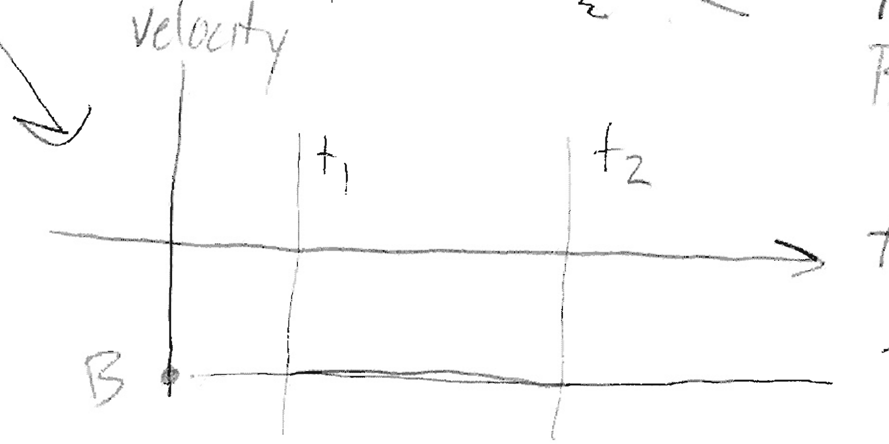
$\bar{x} = A$

$x = A + Bt$



$$\bar{x} = \frac{1}{2}(A + Bt_1 + A + Bt_2)$$
$$= A + B \cdot \left(\frac{t_1+t_2}{2}\right)$$

slope always B

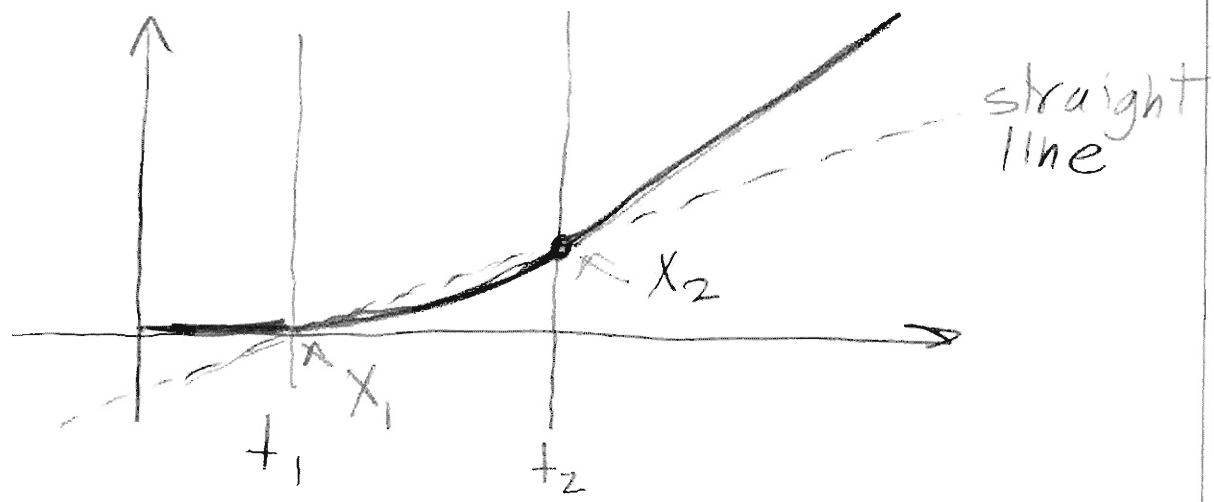


$\bar{v} = B$

acceleration = 0

ALL "EASY"

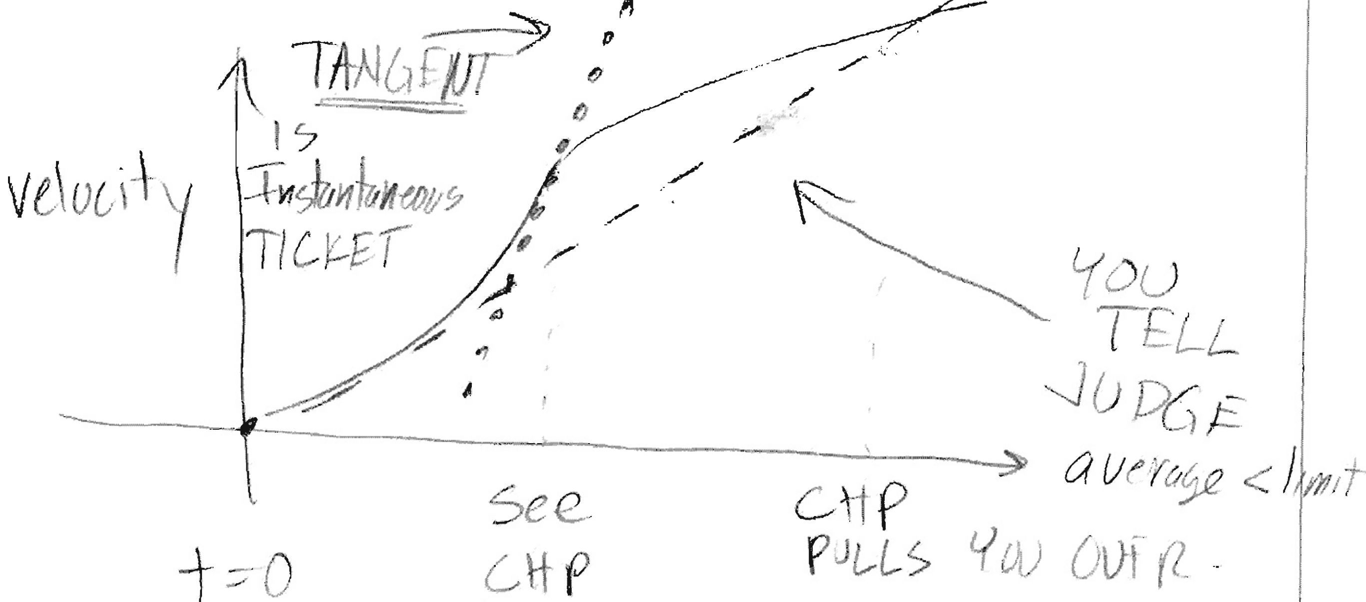
HARDER



$\bar{v}$: that velocity that gets you from x_1 to x_2 with no acceleration... "straight line"

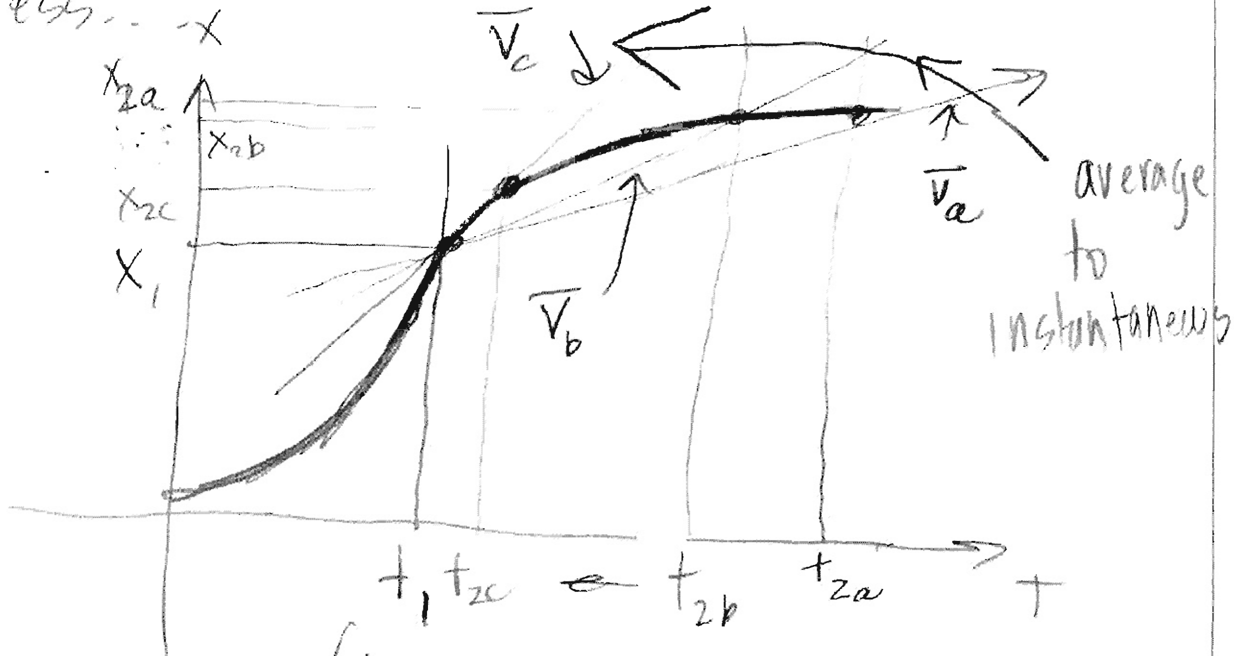
$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1}$$

Instantanews velocity gets you a ticket!



get on freeway, SLOW DOWN.

FORMALLY, tangent defined by a limiting process...



(see CHP)

$$\bar{v}_a = \frac{x_{2a} - x_1}{t_{2a} - t_1}$$

$$\bar{v}_b = \frac{x_{2b} - x_1}{t_{2b} - t_1}$$

$$\bar{v}_c = \frac{x_{2c} - x_1}{t_{2c} - t_1}$$

$$v(t_1) = \lim_{t_2 \to t_1} \frac{x_2(t_2) - x_1(t)}{t_2 - t_1}$$

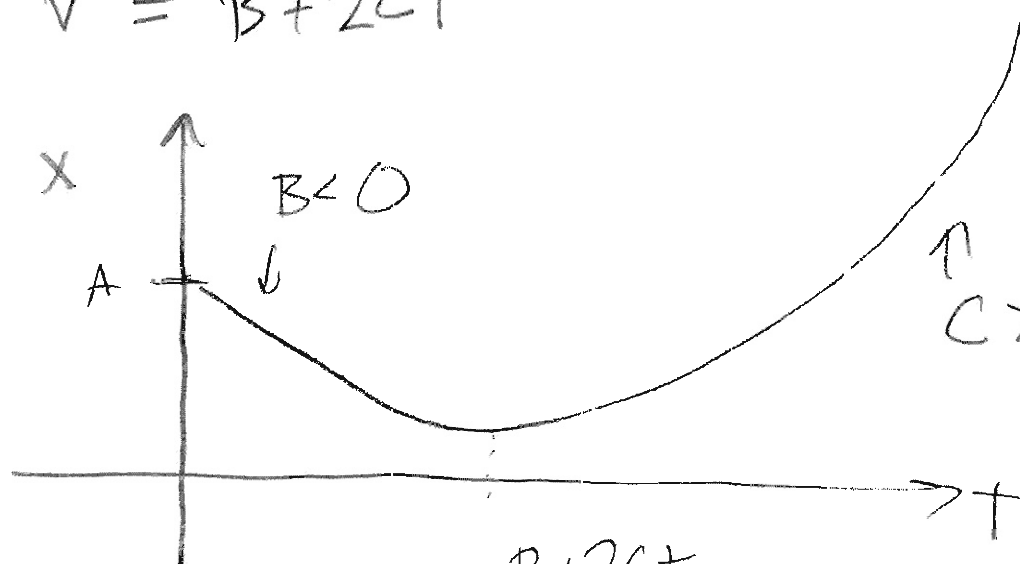
$$v = \frac{dx}{dt} = x' = \dot{x}$$

"CLASSICS"

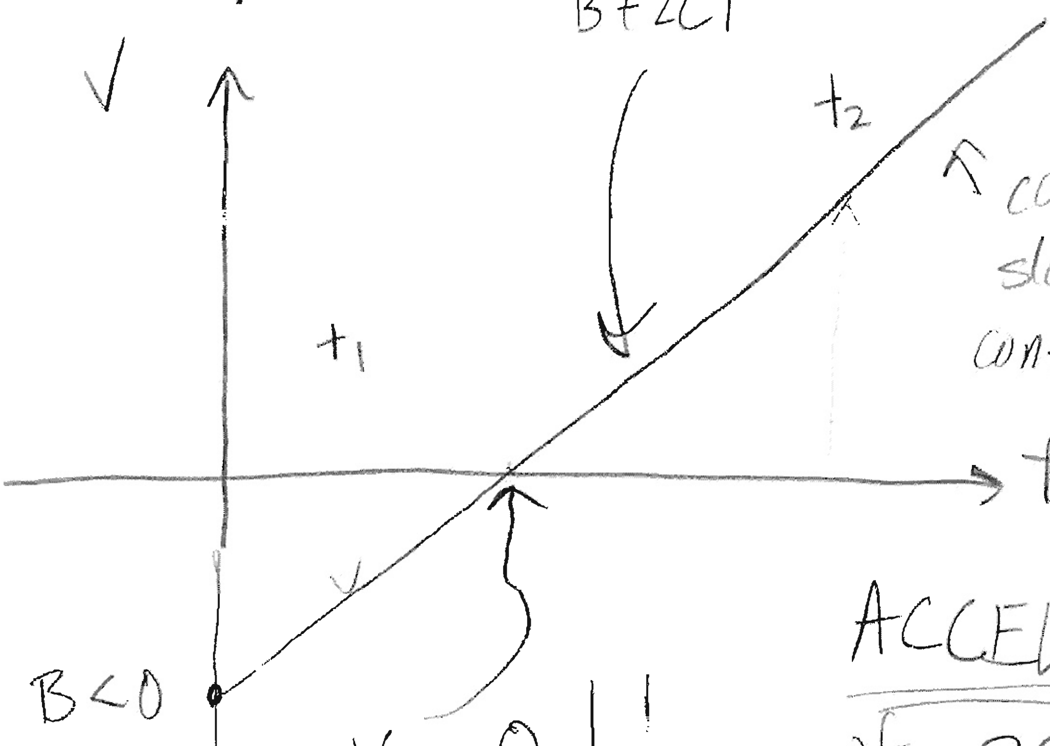
$$x(t) = A + Bt + Ct^2$$

$$V = 0 + B + 2Ct = \frac{dx}{dt}, \; x', \; \dot{x}$$

$$V = B + 2Ct$$



B+2Ct



constant slope means constant

ACCELERATION of $2C > 0$ always here

V = 0 !!

"pause"

Calculate $\bar{v}$ between t_1 & t_2 ?

$$\frac{1}{2}\left[B + 2Ct_1 + B + 2Ct_2\right]$$

$$\bar{v} = B + C(t_1 + t_2)$$

Displacement $= \bar{v}(t_2 - t_1)$

$$= B(t_2 - t_1) + C(t_1 + t_2)(t_2 - t_1)$$

$$s = B(t_2 - t_1) + C(t_2^2 - t_1^2)$$

All this week's problems could now be done... one exception...

$$x(t) = Dt^n$$

$$\frac{dx}{dt} = x' = \dot{x} = nDt^{(n-1)}$$

TOPS.