Displacement: free vector
Position: bound vector

Displacement: start \((x_1, y_1, z_1)\) end \((x_2, y_2, z_2)\)

\[ \vec{S} = (x_2 - x_1) \hat{i} + (y_2 - y_1) \hat{j} + (z_2 - z_1) \hat{k} \]

**NOTE:** \(\alpha\) goes from \((0, 0, 0)\) m to \((1, 2, 3)\) m

\[ \vec{S}_\alpha = (1 + 2\hat{j} + 3\hat{k}) \text{ m} \]

\(\beta\) goes from \((-3, 8, 17)\) to \((-2, 10, 20)\)

\[ \vec{S}_\beta = (\hat{i} + 2\hat{j} + 3\hat{k}) \]

\[ \vec{S}_\alpha = \vec{S}_\beta \] starting point not important for displacement

The Position Vector

\[ x\hat{i} + y\hat{j} + z\hat{k} \]

\[ \vec{r}_2 = \vec{r}_1 - \vec{S}_\alpha \]

\[ \vec{r}_2 = 1\hat{i} + 2\hat{j} + 3\hat{k} \]
\[ \vec{r}_1^\beta = -3\hat{x} + 8\hat{y} + 17\hat{z} \]
\[ \vec{r}_2^\beta = -2\hat{x} + 10\hat{y} + 20\hat{z} \]

- Position vectors depend on where you put the origin of your coordinate system ("bound" vector).
- Displacement vectors do not change a bit.

---

**Motion in one dimension (1.6)**

<table>
<thead>
<tr>
<th>0</th>
<th>1 m</th>
<th>2 m</th>
<th>3 m</th>
<th>4 m</th>
<th>X</th>
</tr>
</thead>
</table>

- "Point" particle.
- If massive, applies to its center of mass.
- Suppose it sits at, say, \( x = 2 \text{ m} \) \( \text{FOREVER} \) (all time).

\[ x(t) = 2 \text{ m} = A \quad (A = 2 \text{ m}) \]

plot that...
slope (rise over run)

\[ x = \frac{x_2 - x_1}{t_2 - t_1} \]

\[ x = A + Bt \]

graph

now imagine

\[ x(t) = A + Bt \]

("constant velocity")

(A can be < 0)

\[ x(t) = A \]

("constant position")
Imagine \( t_2 \rightarrow t_1 \) ... get \( B \) for slope no matter how close \( t_2 \) gets to \( t_1 \).

\[
B: \begin{cases} 
\text{can be} & < 0 \\
\text{can be} & > 0
\end{cases}
\]

\( B \) is a 1-d vector called \textit{velocity}.

\(|B| \) called \textit{speed} more fin in 2-d.

\textit{Constant speed} \neq \textit{constant velocity}

give example
Einstein: If you were moving with constant velocity, without external influence (like, air...) you see all the same laws of physics as someone else moving with a different constant velocity. Relativity.

Acceleration = change in velocity

is where interesting stuff happens particularly in 7.1-6 (l-a or now)

Best example: start from rest (x=0, say)
get to constant velocity must have change, acceleration

\[ \begin{align*}
\text{VELOCITY} & \quad \text{ACCELERATION} \\
\text{CURVE} & \quad \text{VELOCITY (positive)} \\
+1 & \quad +2
\end{align*} \]

Visually:

\[ \begin{align*}
+ & \text{ acceleration (holds water)} \\
- & \text{ acceleration (dumps water)}
\end{align*} \]
Average, Instantaneous
Position
Velocity
Acceleration

in a time interval \( t_1 \) to \( t_2 \)

\[
\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} x(t) \, dt
\]

\[
\bar{x} = \frac{1}{2} \left( A + Bt_1 + A + Bt_2 \right)
\]

\[
= A + B \cdot \left( \frac{t_1 + t_2}{2} \right)
\]

slope always

\[
\bar{v} = B
\]

Acceleration = 0

ALL "EASY"
HARDER

\[ \overrightarrow{V} \text{: that velocity that gets you from } x_1 \text{ to } x_2 \text{ with no acceleration... straight line} \]

\[ \overrightarrow{V} = \frac{x_2 - x_1}{t_2 - t_1} \]

Instantaneous velocity gets you a ticket!

Velocity

TANGENT

IS

Instantaneous TICKET

\[ t = 0 \]

see CHP pulls you over -

get on freeway, SLOW DOWN!

YOU TELL JUDGE AVERAGE < LIMIT
FORMALLY, tangent defined by a limit is
a process.

\[ V_a = \frac{x_{2a} - x_1}{t_{2a} - t_1} \]

\[ V_b = \frac{x_{2b} - x_1}{t_{2b} - t_1} \]

\[ V_c = \frac{x_{2c} - x_1}{t_{2c} - t_1} \]

\( \sqrt{t_1} = \lim_{t_2 \to t_1} \frac{x_2(t_2) - x_1(t)}{t_2 - t_1} \)

\[ V = \frac{dx}{dt} = x' = x \]
"CLASSICS"

\[ x(t) = A + Bt + Ct^2 \]

\[ V = 0 + B + 2Ct \frac{dx}{dt} \]  
\[ x' = x'' \]

\[ V = B + 2Ct \]

\[ \begin{array}{c}
\text{\( B < 0 \) down} \\
\text{\( A \) up} \\
\text{\( C > 0 \) up} \\
\text{\( \text{constant slope means constant} \)} \\
\text{\( +2 \)}
\end{array} \]

\[ \text{ACCELERATION of 2C > 0 always here} \]

\[ \text{pause} \]
Calculate $\bar{V}$ between $t_1$ and $t_2$?

$$\frac{1}{2} \left[ B + 2Ct_1 + B + 2Ct_2 \right]$$

$$\bar{V} = B + C(t_2 - t_1)$$

**Displacement**

$$= \bar{V}(t_2 - t_1)$$

$$= B(t_2 - t_1) + C(t_1 + t_2)(t_2 - t_1)$$

$$S = B(t_2 - t_1) + C\left(t_2^2 - t_1^2\right)$$

**All this week's problems could now be done... one exception...**

$$x(t) = D + n$$

$$\frac{dx}{dt} = x' = x = n D^{(n-1)}$$