Physics 20

Vectors

Quantities with both direction and magnitude.

Geometric Viewpoint:
like compass, straight-edge, "Side-Angle-Side"

Algebraic Viewpoint:
need coordinate axes... two different observers choose different axes, but knowing the geometry means results generally should not depend on that choice.

Tip: when you need axes, choose the one that makes the problem easiest!

Conceptually, # of dimensions is crucial.

1 dimension... # line, simple, illustrative

what origin?
2 dimensions: most problems in 2D, what orientation?

3 dimensions: rarer in 2D, begin to discuss

The first famous vector... displacement

\[ \text{start} \rightarrow \text{end} \]

\[ \text{tip} \rightarrow \text{base} \]

Imagine a "bug" begins, say, at \( t=0 \), at the "start" location, then wanders to the right, back to left, etc, finally ending at \( t=T \) to the left, at "end".

The direction of the displacement vector is from the \text{start} to the \text{end}.

All the intermediate wandering is irrelevant for the displacement. The vector displacement \( \vec{s} \) is shown above.

Geometrically, that is pretty much the story, in 1-D! Worth making this point: the
Location of the beginning of a displacement vector is irrelevant. The following two displacements are equal:

\[ \vec{S}_1 \quad \rightarrow \quad \vec{S}_2 \]

\[
\begin{align*}
| \vec{S}_1 | &= | \vec{S}_2 | \\
\text{Length} &= \text{Length} \\
\Rightarrow | \vec{S}_1 | &= | \vec{S}_2 | \quad \text{(magnitude)}
\end{align*}
\]

To be equal, \( | \vec{S}_1 | = | \vec{S}_2 | \) (magnitude)

right to right to left

\( \vec{S}_1 \) and \( \vec{S}_2 \) are in right to left direction.

Displacement is a "FREE" vector.

opposite: "BOUND" vector... like Forces when computing torques

Multiplication by a scalar \((1,-,0,+ \text{ same})\)

\[
\begin{align*}
b \vec{S}_1 \quad \rightarrow \quad b \vec{S}_1 \\
| b \vec{S}_1 | &= b \times | \vec{S}_1 | \\
\text{flipped} &\quad \text{in} \quad \text{direction} \quad \text{when} \quad b < 0
\end{align*}
\]
$\vec{S}_1$

$\vec{S}_1 = -2 \vec{S}_1$

$b = -2$

Adding, multiplying, even dividing 1-d vectors... just like real #s, better wait until 2 dimensions, 3 dimensions, etc.

Base Vector + Components

A base vector has no dimensions, only a direction. In one dimension, the base vector is usually called $\hat{i}$. (i-hat) (sometimes called $\hat{x}$)

$$\vec{S}_1 = \text{(number, units of length)} \cdot \hat{i}$$

$\vec{S}_1 = \hat{i} \cdot S_1$

The (in 1-d) component of $\vec{S}_1$!

To really specify $S_1$, need a system of units... METRIC SYSTEM! METERS!
Into the Second Dimension brings
- addition of vectors that is more subtle than simply real #s
- a type of multiplication that is also more subtle... scalar or dot product. #

Into the Third Dimension brings
- two more types of multiplication
  1) Vector or Cross Product, which DOES NOT COMMUTE! #
  2) Tensor Product (not actually covered in Physics 20) #

Generalize... $\mathbf{A}, \mathbf{B}, \mathbf{C}$ vectors 2-d for now.

Example $\mathbf{A}$ $\mathbf{B}$

for FREE vectors, (most common), as long as lengths same, directions same, $\mathbf{A} = \mathbf{B}$
A's unit vector is \( \hat{A} \) (A-hat).

\[ \hat{A} = \frac{\vec{A}}{|\vec{A}|} \]

"knows direction of \( \vec{A} \)"

\[ b = -\frac{1}{2}, \quad b \cdot \vec{A} \rightarrow \text{draw that} \]

\[ \Downarrow b\vec{A} \rightarrow \text{direction flipped half as long} \]

Vector Addition in 2 or more dimensions for FREE vectors

"Geometric" addition... can translate (without rotation) \( \vec{A} \) or \( \vec{B} \).

\[ \vec{A} + \vec{B} \]

"tip to tail"

Note: \( \vec{B} + \vec{A} \) gives same result! \( \vec{A} + \vec{B} \) "Resultant"
Vector Addition is Commutative
(see the parallelogram on the previous page)
\[ \vec{A} + \vec{B} = \vec{B} + \vec{A} \]

Vector Subtraction is addition of the negative:
\[ \vec{A} - \vec{B} = \vec{A} + (-\vec{B}) \]

"Triangle Inequality" not in text

In 1 dimension, adding vectors is like adding real numbers,

\[ |\vec{A} + \vec{B}| = \begin{cases} 
|\vec{A}| + |\vec{B}| & \text{(relative sign } \vec{A} + \vec{B} \text{ same)} \\
|\vec{A}| - |\vec{B}| & \text{(relative sign } \vec{A} + \vec{B} \text{ different)} 
\end{cases} \]

THINK: (-dim, what conditions give \( |\vec{A} - \vec{B}| = 0 \)
THINK: In 2 (or 3) dimensions, This Changes

\[ |\overrightarrow{A} + \overrightarrow{B}| \text{ only} = |\overrightarrow{A}| + |\overrightarrow{B}| \]

when \( \overrightarrow{A} \) and \( \overrightarrow{B} \) are parallel (like)

\[ \overrightarrow{B} \]

\[ \overrightarrow{A} \]

\[ \overrightarrow{A} + \overrightarrow{B} \]

\[ |\overrightarrow{A} + \overrightarrow{B}| \text{ only} = |\overrightarrow{A}| - |\overrightarrow{B}| | \]

when \( \overrightarrow{A} \) and \( \overrightarrow{B} \) are antiparallel

\[ \overrightarrow{A} \]

\[ \overrightarrow{B} \]

In general, in 2 or more dimensions,

\[ |\overrightarrow{A} - \overrightarrow{B}| \leq |\overrightarrow{A} + \overrightarrow{B}| \leq |\overrightarrow{A}| + |\overrightarrow{B}| \]

"Triangle Inequality"