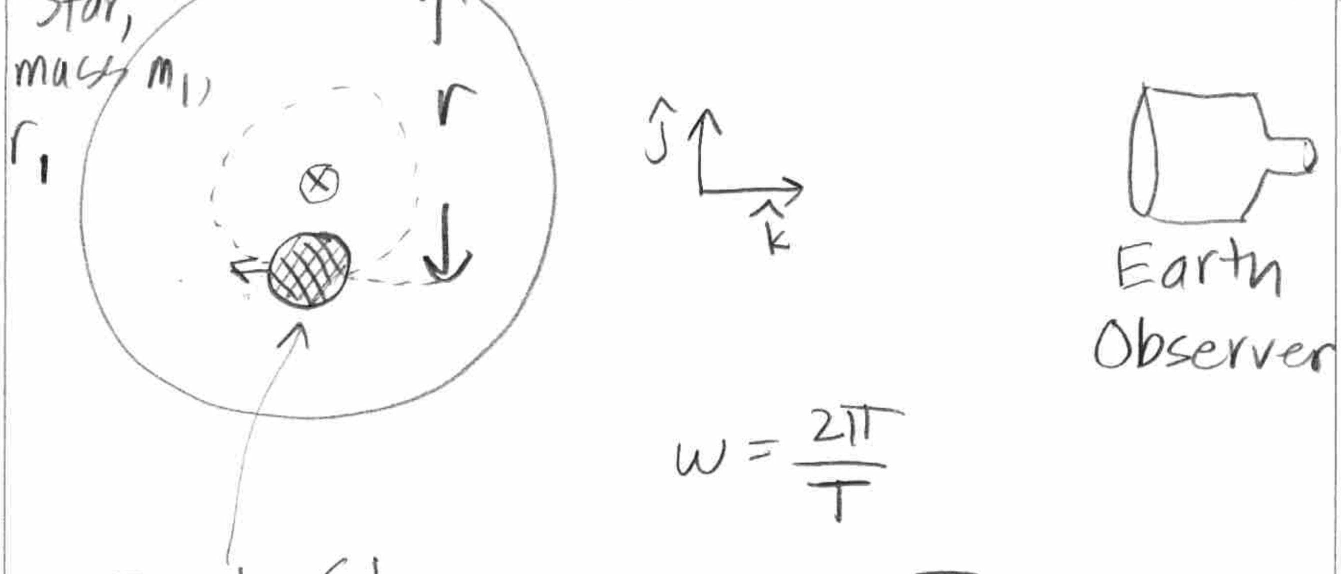
1 (a)

$\vec{V}_1 = V_{01}\left(\cos(\omega t)\hat{k} - \sin(\omega t)\hat{j}\right)$



$\omega = \frac{2\pi}{T}$

Dark Star, mass m_2, radius r_2

T = period

$$r_1 = f_2 r = \frac{m_2}{m_1+m_2} r$$

$$r_2 = f_1 r = \frac{m_1}{m_1+m_2} r$$

Figure shows $\hat{k}$ component of <u>bright star's</u> velocity, showing $V_{01} \cong 140$ km/s, $T = 33.5$ days.

From discussion in class,

$$\vec{r} \to \vec{r}_1 - \vec{r}_2, \quad \dot{\vec{r}} = \vec{V} \neq \vec{V}_1, \vec{V}_2$$

$$\mu \ddot{\vec{r}} = -\frac{m_1 m_2}{m_1+m_2}\frac{V^2}{r} = -\frac{G m_1 m_2}{r^2}$$

$$m_1 + m_2 = \frac{rv^2}{G}$$ } must put in terms of observed quantities, v_{01} & T

since $\dot{\vec{r}} = \vec{v}$, $\vec{v}_1 = \dot{\vec{r}}_1 = f_2 \dot{\vec{r}} = f_2 \vec{v}$

$$\underline{v = \frac{1}{f_2} v_{01}}$$ (assume circular)

$$vT = 2\pi r$$

$$r = \frac{vT}{2\pi} = \frac{1}{f_2}\frac{v_{01}T}{2\pi}$$

$$m_1 + m_2 = \frac{1}{G} \cdot \frac{1}{f_2}\frac{v_{01}T}{2\pi} \cdot \frac{1}{f_2^2} v_{01}^2$$

$$= \frac{1}{G}\frac{v_{01}^3 T}{f_2^3 \, 2\pi} = \frac{1}{G}\frac{(m_1+m_2)^3}{m_2^3}\frac{v_{01}^3 T}{2\pi}$$

$$\frac{m_2^3}{(m_1+m_2)^3}(m_1+m_2) = \frac{v_{01}^3 T}{2\pi G}$$

(a) $$\boxed{\frac{m_2^3}{(m_1+m_2)^2} = \frac{v_{01}^3 T}{2\pi G}}$$

(b) Point is ....

$$\frac{m_2^3}{(m_1+m_2)^2} = \frac{m_2}{\left(1+\frac{m_1}{m_2}\right)^2} < m_2$$

if $\dfrac{m_2}{\left(1+\frac{m_1}{m_2}\right)^2} > 3$ solar masses,

m_2 should be a Black Hole..

$$V_{01} = 140 \ \frac{km}{s} = 1.4 \cdot 10^2 \cdot 10^3 \ m/s$$
$$= 1.4 \cdot 10^5 \ m/s$$

$$T = 33.5 \text{ days} = 2.89 \cdot 10^6 \ s$$

$$\frac{V_{01}^3 T}{2\pi G} = \frac{(1.4 \cdot 10^5)^3 \cdot (2.89 \cdot 10^6)}{2 \cdot \pi \cdot 6.67 \cdot 10^{-11}} \ \frac{\frac{m^3}{s^3} \cdot s}{\frac{m^3}{kg}\frac{1}{s^2}}$$

$$= 1.89 \cdot 10^{31} \ kg$$

$$M_\odot = 1.99 \cdot 10^{30} \ kg$$

$$\frac{\left(\frac{V_{10}^3 T}{2\pi G}\right)}{M_\odot} = 9.5 > 3,$$

GRS 1915+105

"new physics"

(c) Call $\mu_2 = \frac{m_2}{m_\odot}$ $\quad \mu_1 = \frac{m_1}{m_\odot} = 1.2$ $\quad \tilde{\mu}_2 = \frac{\frac{v_{01}^3 T}{2\pi G}}{m_\odot} = 9.52$

then $\frac{\mu_2^3}{(\mu_1+\mu_2)^2} = \tilde{\mu}_2$

$$\mu_2^3 - \tilde{\mu}_2\mu_2^2 - 2\tilde{\mu}_2\mu_1\mu_2 - \tilde{\mu}_2\mu_1^2 = 0$$

Used: http://www.1728.com/cubic.htm

plugging in:

$$\mu_2^3 - 9.52\mu_2^2 - 22.86\mu_2 - 13.71 = 0$$

$$\mu_2 = 11.6 \quad \text{or}$$

$$\boxed{m_2 = 11.6\, M_\odot}$$

(d) In addition, there is evidence that the system is tilted by 20° with respect to us:

$V_{01} = V_{1tot} \cdot \cos\theta$

$V_{1tot} = \frac{V_{01}}{\cos\theta}$ $\qquad \theta = 20^\circ$

now $\tilde{\mu}_2 \Rightarrow \frac{\frac{V_{1tot}^3 T}{2\pi G}}{m_\odot} = 11.48$

now $\mu_2^3 - \tilde{\mu}_2 \mu_2^2 - 2\tilde{\mu}_2 \mu_1 \mu_2 - \tilde{\mu}_2 \mu_1^2 = 0$

$$\mu_2^3 - 11.48\mu_2^2 - 27.55\mu_2 - 16.53 = 0$$

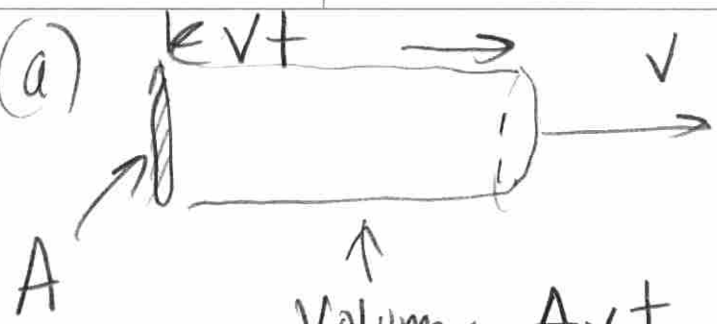
$$\mu_2 = 13.6$$

$$m_2 = \mu_2 m_\odot = 13.6\, m_\odot$$

A proper analysis, with errors, gives

$$m_2 = (14 \pm 4)\, m_\odot$$

(2.) (a)



Volume = Avt

$$t = \frac{V}{Av} \Leftarrow \text{of volume rocket chamber}$$

$$\boxed{\begin{aligned} t_1 &= \frac{2 \cdot 10^{-3}\ m^3}{2 \cdot 10^{-4}\ m^2 \cdot 20\ m/s} = \frac{1}{2}\ s \\ t_2 &= \frac{2 \cdot 10^{-3}\ m^3}{4 \cdot 10^{-4}\ m^2 \cdot 20\ m/s} = \frac{1}{4}\ s \end{aligned}}$$

(b) Eq on p. 139

$$v_f = u \ln\left(\frac{M_0}{M_f}\right) - gt$$

2 kg = fuel mass
0.1 kg = rocket "

$$\ln\left(\frac{M_0}{M_f}\right) = \ln\left(\frac{2+0.1}{0.1}\right)$$

$$= 3.04$$

$$u \ln\left(\frac{M_0}{M_f}\right) = 20 \cdot 3.04 = 60.9\ m/s$$

$$\boxed{\begin{aligned} &\text{Case 1: } v_f = 60.9 - 9.8 \cdot \tfrac{1}{2} = 55.0\ m/s \\ &\text{Case 2: } v_f = 60.9 - 9.8 \cdot \tfrac{1}{4} = 57.5\ m/s \end{aligned}}$$

(c) After boost phase, added distance h,

$$v^2 = 2gh$$

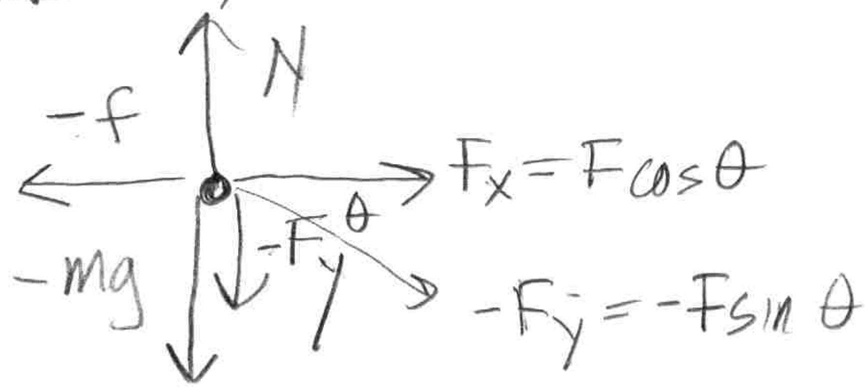
$$h = \frac{v^2}{2g}$$

Case	boost phase $\frac{1}{2}v_f t$	ballistic phase	total
#1	$\frac{1}{2} \cdot 55 \cdot \frac{1}{2} = 13.8\,m$	$\frac{v^2}{2g} = \frac{55^2}{2 \cdot 9.8} = 154.4\,m$	168.2 m
#2	$\frac{1}{2} \cdot 57.5 \cdot \frac{1}{4} = 7.2\,m$	$\frac{v^2}{2g} = \frac{57.5^2}{2 \cdot 9.8} = 168.5\,m$	175.6 m

so, brief boost phase attains higher elevation. (#2)

(3)

Make force diagram... since constant speed, net force is 0.



$$N - F_y - mg = 0$$

$$N - F\sin\theta - mg = 0$$

$$-f = -\mu_k N$$

$$F_x - f = 0 = F\cos\theta - \mu_k N = 0$$

$$F = \frac{\mu_k N}{\cos\theta}$$

$$N - \frac{\mu_k N \sin\theta}{\cos\theta} = mg$$

$$N = \frac{mg}{1 - \mu_k \tan\theta}$$

$$F = \frac{\mu_k N}{\cos\theta} = \frac{mg}{\cos\theta(1-\mu_k \tan\theta)}$$

$$\boxed{F = \frac{mg}{\cos\theta - \mu_k \sin\theta}}$$

<u>Work</u>: $F_x \cdot d = F\cos\theta \cdot d$

$$\boxed{W = \frac{mg\cos\theta\, d}{\cos\theta - \mu_k \sin\theta}}$$

numerically: $m = 26.6\text{ kg}$

$g = 9.8\ m/s^2$

$\theta = 32^\circ$

$d = 9.54\text{ m}$

$\mu_k = 0.21$

$$W = \frac{26.6 \cdot 9.8 \cdot \cos(32^\circ) \cdot 9.54}{\cos(32^\circ) - 0.21 \sin(32^\circ)}$$

$$\boxed{W = 2860 \text{ Joules}}$$

TOPS