Physics 20 Problem Set 9

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due Monday, November 29, by 5pm
to the Physics 20 Boxes in Broida Hall's Lobby

Course Announcments: Please return your midterm for checking when you turn in this problem set, or bring your midterm to class. These problems pertain to the ninth and tenth week's lectures, and the corresponding reading is pages 152-168 of KK Chapter 4, and pages 131-138 of RHK4 Chapter 7. There will be no PSR fellows on Wed. and Thurs. 11/24 and 11/25, but they should be there Sunday 11/28.

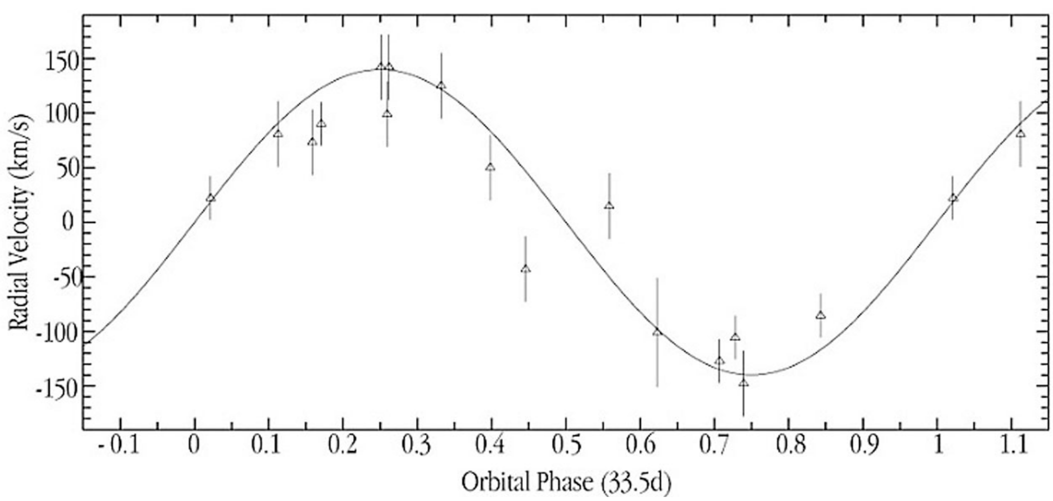

Figure 1: Problem 1.

1. Our work on two-body systems enable us to understand the evidence for stellar black holes. One of the strongest cases for a stellar black hole is the binary system GRS 1915+105. GRS stands for GRanat Source; Granat was a Soviet/French/Danish/Bulgarian satellite that first discovered this system in 1992; 1915+105 gives the location of the source in astrophysical coordinates. In practical terms, GRS 1915+105 is in the constellation Aquila (the Eagle), which is easy to see in the summer sky.

   All of the data needed for this problem is in Fig. 1. The horizontal axis of that figure is time, and you can see that the pattern repeats itself after $T = 33.5\,\text{days}$; T is the period of the binary's orbit. The binary system consists of a visible star and a star that does not emit light; call the visible star #1 and the dark star (the black hole) #2. It is possible to measure the component of #1's velocity along a line connecting Earth to the star using the so-called 'Doppler Shift,' and that measurement comprises the vertical axis of Fig. 1. The curve represents a sinusoidal fit to the data; the data is shown with the trianglular data points with error bars. The amplitude of the sinusoid is $v_{01} = 140\,\text{km/s}$.

For this problem, let's assume that the orbits of the visible star and the dark star are circular, and that the Earth lies exactly in the plane of their orbit. With a circular orbit so defined, it should be clear to you why the component of the visible star's velocity along the line to the Earth is sinusoidal as shown in Fig 1.

(a) Use the law of gravitation and the equations that describe the internal forces in the two-body problem to show:

$$\frac{m_2^3}{(m_1+m_2)^2} = \frac{v_{01}^3 T}{2\pi G}$$

(b) Now show that the following inequality holds:

$$m_2 > \frac{m_2^3}{(m_1+m_2)^2}.$$

Thus, numerically evaluate the lower limit on m_2 that arises from the measurements of T and v_{01}, and put that lower limit in terms of the mass $M_\odot$ of our Sun, which is $M_\odot = 1.99\times 10^{30}$ kg. Generally, if $(M/M_\odot) > 3$, then the dark star must have 'new physics,' most likely a black hole. If $(M/M_\odot) > 10$, the case for a black hole is extremely strong. What do you get for $(m_2/M_\odot)$ for GRS 1915+105?

(c) Independent evidence indicates $m_1 = 1.2M_\odot$. Use this value to re-evaluate $(m_2/M_\odot) \equiv \mu_2$. You will have to solve a cubic equation, of the form:

$$A\mu_2^3 + B\mu_2^2 + C\mu_2 + D = 0.$$

It is easy to find, on the web, cubic equation solvers, if you enter the numerical values for A, B, C, and D. One such solver is at the URL: `http://www.1728.com/cubic.htm`.

(d) Actually, the best current value for μ_2 is 13.6. Qualitatively, what simple physical effect that we are able to calculate would cause an even bigger value of μ_2?

2. A water rocket holds 2 kg of water in a volume of 2 liters $= 2\times 10^{-3}\,\mathrm{m}^3$; the mass of the rocket is 0.1 kg. The water is pressurized by a hand pump; when a switch is thrown, the water rushes out at a speed *relative to the rocket* of $u = 20$m/s, downward, shooting the rocket up vertically. You can attach either one of two available nozzles, the first with area $A_1 = 2\times 10^{-4}\,\mathrm{m}^2$, or the second with area $A_2 = 4\times 10^{-4}\,\mathrm{m}^2$.

(a) How long, numerically, does it take to eject all of the water rocket's water, for the two nozzle cases? This is the time of the 'boost phase'.

(b) Assuming the rocket starts from rest, what is the final velocity v_f of the rocket, numerically, in the two nozzle cases?

(c) *Approximate* the distance travelled upward during the 'boost phase' of the rocket as $(1/2)v_f t$, where t is the time of the boost phase. For the 'ballistic' phase of the rocket flight, where no water is ejected, you know how to estimate the elevation. For the two cases of nozzles, numerically, what maximum *total* elevations that the rocket can attain for the two cases? Which case carries the rocket higher?

3. A worker pushes a 58.7 lb block ($m = 26.6$ kg) a distance of 31.3 ft (=9.54 m) along a level floor at a constant speed with a force directed 32° below the horizontal. The coefficient of kinetic friction is 0.21. How much work did the worker do on the block? (RHK4 7.6).