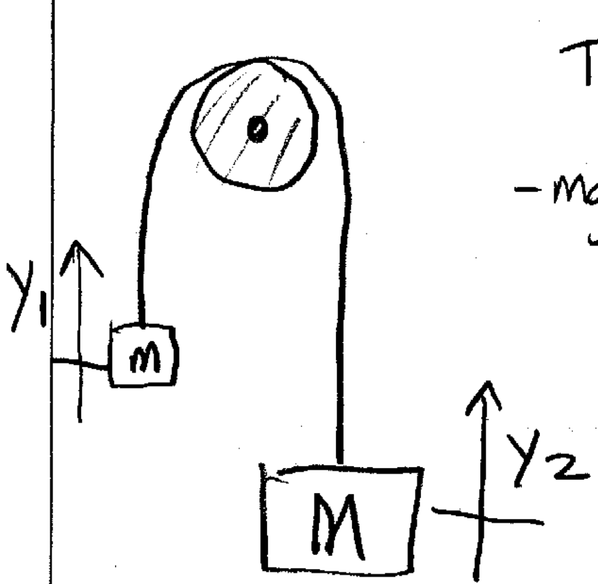
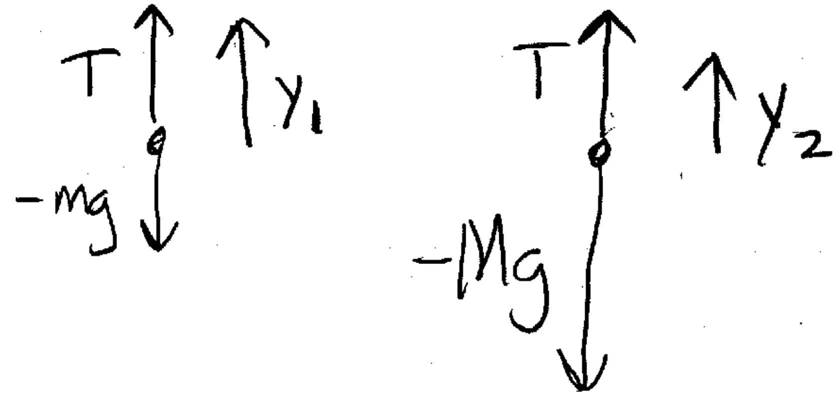
1 KK 2.5





$$T - mg = m\ddot{y}_1$$

$$T - Mg = M\ddot{y}_2$$

string doesn't stretch $\quad -\ddot{y}_1 = \ddot{y}_2$

want T, $\ddot{y}_2$

$$T - mg = -m\ddot{y}_2$$

$$\ddot{y}_2 = -\frac{T}{m} + g$$

$$T - Mg = M\left(-\frac{T}{m} + g\right)$$

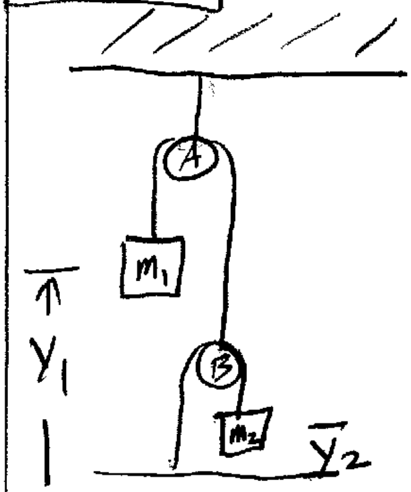
$$\left(1 + \frac{M}{m}\right)T = 2Mg \qquad M = 2m$$

$$\boxed{T = \frac{2M}{1 + \frac{M}{m}}\, g = \frac{2Mm}{m+M}\, g} = \frac{4m^2}{3m} g = \frac{4}{3} mg = \frac{2}{3} Mg$$

$$\ddot{y}_2 = -\frac{m}{3m} g = -\frac{g}{3}$$

$$\boxed{\ddot{y}_2 = -\frac{T}{m} + g = \left(\frac{-2M}{m+M} + 1\right) g = \left(\frac{m-M}{m+M}\right) g}$$

② 2.13



$$2\ddot{y}_1 = -\ddot{y}_2$$

think: if y_1 goes up by Δ, pulley B goes down by Δ, leaving 2Δ of slop for m_2.

mass m_2: $\uparrow T_2$, $\downarrow -m_2 g$

$$T_2 - m_2 g = m_2 \ddot{y}_2$$

Pulley B: $\uparrow T_1$, $\downarrow -T_2$, $\downarrow -T_2$

$$T_1 - 2T_2 = \cancel{m}\cdot(-\ddot{y}_1)$$

$m = 0$

$$T_1 - 2T_2 = 0$$

mass m_1: $\uparrow T_1$, $\downarrow m_1 g$

$$T_1 - m_1 g = m_1 \ddot{y}_1$$

want to solve for $\ddot{y}_1$

$$T_2 - m_2 g = -2m_2\ddot{y}_1$$

$$T_1 = 2T_2 = 2m_2(g - 2\ddot{y}_1)$$

$$2m_2(g-2\ddot{y}_1) - m_1 g = m_1\ddot{y}_1$$

$$(2m_2 - m_1)g = (4m_2 + m_1)\ddot{y}_1$$

$$\boxed{\ddot{y}_1 = \frac{2m_2 - m_1}{4m_2 + m_1}\, g}$$

HINT $\Rightarrow$ $m_1 = m_2$, $\dfrac{2m_1 - m_1}{4m_1 + m_1} g = \dfrac{1}{5} g$

③ 1.17 $\dot{r} = 4\ m/s$ $\dot{\theta} = 2\ rad/s$ 3/7

(a) $\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$

$\dot{r} = 4\ \frac{m}{s}$, $\underbrace{r = 3m \quad \dot{\theta} = 2\ rad/s}_{6}$

$$\boxed{v = |\vec{v}| = \sqrt{4^2 + 6^2} = \sqrt{16+36} = \sqrt{52}\ m/s}$$

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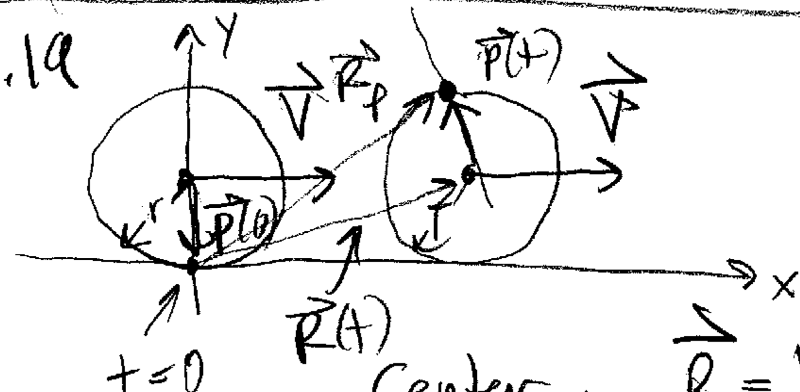
(b) $\vec{a} = (\underbrace{\ddot{r}}_{0} - r\dot{\theta}^2)\hat{r} + (r\underbrace{\ddot{\theta}}_{0} + 2\dot{r}\dot{\theta})\hat{\theta}$

$\dot{r} = $ constant $\qquad \dot{\theta} = $ constant

$$\vec{a} = -r\dot{\theta}^2\hat{r} + 2\dot{r}\dot{\theta}\hat{\theta}$$
$$= \left(-3\cdot 2^2\hat{r} + 2\cdot 4\cdot 2\,\hat{\theta}\right) m/s^2$$
$$= \left(-12\hat{r} + 16\hat{\theta}\right) m/s^2$$
$$|\vec{a}| = \sqrt{12^2 + 16^2}\; m/s^2$$
$$= \sqrt{(-3\cdot 2^2)^2 + (4\cdot 2^2)^2}\; m/s^2$$
$$= 2^2\underbrace{\sqrt{(-3)^2 + (4)^2}}\; m/s^2$$
$$\boxed{|\vec{a}| = 2\cdot 2\cdot 5\; m/s^2 = 20\; m/s^2}$$

4) 1.19



r = radius of tire

$\cancel{Vt} = r\cancel{\omega t}$

$V = r\omega$

<u>Center</u>: $\vec{R} = Vt\,\hat{\imath} + r\hat{\jmath}$
$= r\omega t\,\hat{\imath} + r\hat{\jmath}$

<u>Pebble</u>: $\vec{p}(t) = -r\sin(\omega t)\hat{\imath} - r\cos(\omega t)\hat{\jmath}$

$$\vec{R}_p(t) = \vec{R}(t) + \vec{p}(t)$$

position $\boxed{\vec{R}_p(t) = r[(\omega t - \sin(\omega t))\hat{\imath} + (1 - \cos(\omega t))]\hat{\jmath}}$

<u>velocity</u>: $\boxed{\vec{V}_p(t) = \frac{d\vec{R}_p}{dt} = r\omega\left[(1 - \cos(\omega t))\hat{\imath} + \sin(\omega t)\hat{\jmath}\right]}$

<u>acceleration</u> $\boxed{\vec{a}_p(t) = \frac{d\vec{V}_p}{dt} = r\omega^2\left[\sin(\omega t)\hat{\imath} + \cos(\omega t)\hat{\jmath}\right]}$

sometimes useful: $1 - \cos(\omega t)$

$$= 1 - \left(\cos^2\left(\frac{\omega t}{2}\right) - \sin^2\left(\frac{\omega t}{2}\right)\right)$$

$$= \sin^2\left(\frac{\omega t}{2}\right) + \sin^2\left(\frac{\omega t}{2}\right)$$

$$1 - \cos(\omega t) = 2\sin^2\left(\frac{\omega t}{2}\right)$$

$$\vec{R}_p(t) = r\left[(\omega t - \sin(\omega t))\hat{\imath} + 2\sin^2\left(\frac{\omega t}{2}\right)\hat{\jmath}\right]$$

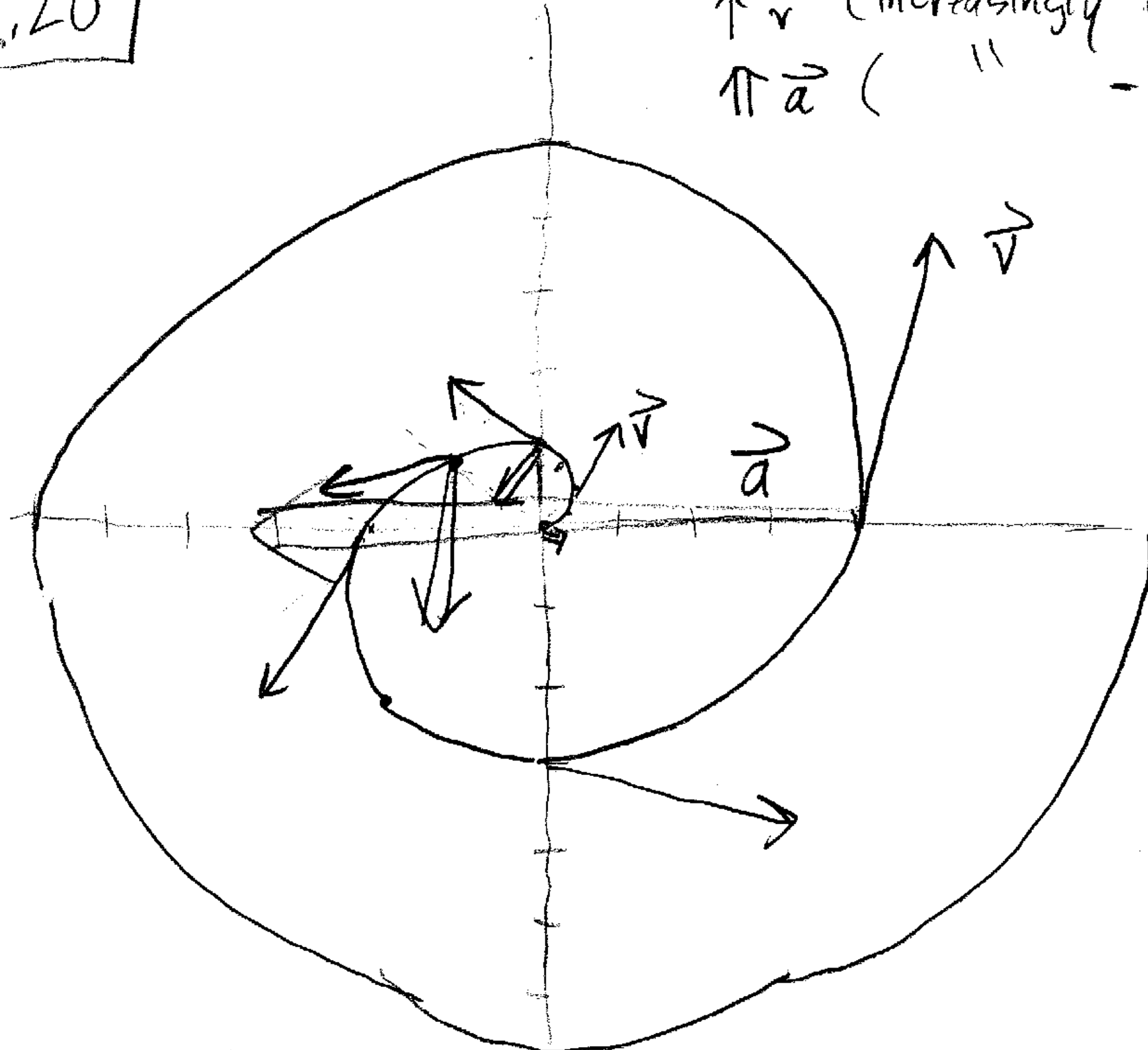
$$\vec{V}_p = r\omega\left[\underbrace{2\sin^2\left(\frac{\omega t}{2}\right)\hat{\imath} + \sin(\omega t)\hat{\jmath}}_{t=0,\ \vec{V}_p(0)=0}\right]$$

$$\vec{a}_p = r\omega^2\left[\sin(\omega t)\hat{\imath} + \cos(\omega t)\hat{\jmath}\right]$$

5) 1,20

$\uparrow \vec{v}$ (increasingly $\hat{\theta}$)

$\Uparrow \vec{a}$ (" $-\hat{r}$)



$\theta = \frac{1}{2}\alpha t^2 \Rightarrow t = \sqrt{\frac{2\theta}{\alpha}}$

$r = A\theta = \frac{1}{2}A\alpha t^2$

$\dot{\theta} = \alpha t = \alpha \cdot \sqrt{\frac{2\theta}{\alpha}}$

$\dot{r} = A\alpha t = A\alpha \cdot \sqrt{\frac{2\theta}{\alpha}}$

$\dot{\theta} = \sqrt{2\alpha\theta}$

$\dot{r} = A\sqrt{2\alpha\theta}$

$\ddot{\theta} = \alpha$

$\ddot{r} = A\alpha$

note $\frac{r}{\theta} = A = \frac{\dot{r}}{\dot{\theta}} = \frac{\ddot{r}}{\ddot{\theta}}$

(a) $\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} = A\sqrt{2\alpha\theta}\,\hat{r} + A\theta\sqrt{2\alpha\theta}\,\hat{\theta}$

$= A\sqrt{2\alpha\theta}\left(\hat{r} + \theta\hat{\theta}\right)$

θ	$\vec{v}\ (A\cdot\sqrt{2\alpha})$	$\vec{a}\ (A\alpha)$
0	0	$\hat{r}$
$\pi/4$	$\sqrt{\frac{\pi}{4}}\left(\hat{r}+\frac{\pi}{4}\hat{\theta}\right)$	$\left(1-2\left(\frac{\pi}{4}\right)^2\right)\hat{r}+5\left(\frac{\pi}{4}\right)\hat{\theta}$
$\pi/2$	$\sqrt{\frac{\pi}{2}}\left(\hat{r}+\frac{\pi}{2}\hat{\theta}\right)$	$\left(1-2\left(\frac{\pi}{2}\right)^2\right)\hat{r}+5\left(\frac{\pi}{2}\right)\hat{\theta}$
$3\pi/4$	$\sqrt{\frac{3\pi}{4}}\left(\hat{r}+\frac{3\pi}{4}\hat{\theta}\right)$	$\left(1-2\left(\frac{3\pi}{4}\right)^2\right)\hat{r}+5\left(\frac{3\pi}{4}\right)\hat{\theta}$
π	$\sqrt{\pi}\left(\hat{r}+\pi\hat{\theta}\right)$	
$3\pi/2$	$\sqrt{\frac{3\pi}{2}}\left(\hat{r}+\frac{3\pi}{2}\hat{\theta}\right)$	
2π	$\sqrt{2\pi}\left(\hat{r}+2\pi\hat{\theta}\right)$	$\left(1-2(2\pi)^2\right)\hat{r}+5(2\pi)\hat{\theta}$

$$\vec{a}=(\ddot{r}-r\dot{\theta}^2)\hat{r}+(r\ddot{\theta}+2\dot{r}\dot{\theta})\hat{\theta}$$

$$=(A\alpha-A\theta\cdot 2\alpha\theta)\hat{r}+\left(A\theta\alpha+2A\sqrt{2\alpha\theta}\sqrt{2\alpha\theta}\right)\hat{\theta}$$

(b) $\vec{a}=A\alpha(1-2\theta^2)\hat{r}+5A\theta\alpha\hat{\theta}$

$=0$ when

$1-2\theta^2=0$

$\theta=1/\sqrt{2}$

$\simeq 0.7\simeq \pi/4$

as $\theta\to\infty$, $\vec{a}\to -A\alpha 2\theta^2\hat{r}$

centripetal

(c) when $\cancel{A\alpha}(1-2\theta^2)=5A\theta\cancel{\alpha}$

$2\theta^2-5\theta-1=0 \Rightarrow \theta=\frac{5\pm\sqrt{25-8}}{4}$

$$\boxed{\theta_\pm=\frac{5\pm\sqrt{17}}{4}}$$