Physics 128 Lecture

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Some assumptions...

- You have taken enough high-school statistics, quantum mechanics, to understand:
  - Mean of a distribution
  - Standard Deviation of a distribution
- Good reference: Bevington/Robinson... first 4 chapters
- You know the Gaussian distribution

\[ p_G = \frac{1}{\sigma \sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( \frac{x - \mu}{\sigma} \right)^2 \right] \]
More...

• You’re not used to the “ad hoc” nature of how experimental physics exploits statistics. Kind of like jazz or composition in music... your textbook learning is like playing from sheet music handed to you.

• Experimental physics: language doesn’t even agree internally, or with academic statisticians.

• **However, in all (particularly experimental) science you must make error estimates. No error estimate... it is not science.**

• Best effort always appreciated. No effort... unacceptable. Mistakes... healthy scientific critique. You’ll need a thick skin... it is not about you, but, the science.

• Experimental science... a drive to the smallest error... sometimes, when ”sensitivity” (like 1/(error size)) passes a threshold, great new discoveries possible.

• Page 14-15 of Bevington... many types of errors. Big categories: “important”, “statistical”, “systematic” are major. Others... “extrinsic” and “intrinsic”. 
Some particle physics measurements over time
All roads lead to.... Gaussian (aka, the central limit theorem)

\{x_i\} = 0’s and 1’s, equal probability; \(N\) picks

\(\bar{x} = \frac{1}{N} \sum x_i \ldots \text{known as the “sample mean”} \)

\(s^2 = \frac{1}{N-1} \sum (x_i - \bar{x})^2 \ldots \text{“sample variance”} \)

Imagine getting \(N\) measurements, computing \(\bar{x}\), and repeating that many times. You’ll get a set \(\{\bar{x}_j\}\).
Central limit theorem

How are \( \{ \bar{x}_j \} \) distributed?
Central limit theorem

How are \( \{ \bar{x}_j \} \) distributed?

As \( N \to \infty \), \( \bar{x}_j \) are distributed about \( \mu \), (\( = 1/2 \) in this example) in a Gaussian distribution with:

\[
\sigma_{\bar{x}} = \frac{S}{\sqrt{N}}
\]

for “arbitrary” distribution of \( x_i \).
Error propagation

Sphere

\[ V = \frac{4\pi}{3} r^3 \]

\[ \bar{r} \pm \sigma_r \quad \text{what is } \sigma_V? \]

\[ \delta V = 3 \frac{4\pi}{3} r^2 \delta r \]

\[ \frac{\delta V}{\sigma_V} \quad \langle \delta r^2 \rangle = 0 \]

\[ \langle \delta V^2 \rangle = 0 \left( \frac{\delta V}{\bar{V}} \right)^2 \frac{\delta r^2}{r^2} \]

\[ \sigma_V^2 = 3 \left( \frac{\delta V}{\bar{V}} \right)^2 \frac{\sigma_r^2}{r^2} \]

\[ \frac{\sigma_V}{\bar{V}} = 3 \frac{\sigma_r}{r} \]

Cylinder

\[ V = \pi r^2 h \]

\[ \bar{r} \pm \sigma_r, \quad \bar{h} \pm \sigma_h \]

\[ \delta V = (2\pi r) \delta r + \pi r^2 \delta h \quad \Rightarrow \langle \delta V \rangle = 0 \]

\[ \frac{\delta V}{\sigma_V} \quad \frac{\delta V}{\delta h} \quad \text{"correlated"} \]

\[ \langle \delta V^2 \rangle = (2\pi \bar{r} \bar{h})^2 \langle \delta r^2 \rangle + 2(2\pi \bar{r} \bar{h}) \langle \delta r \delta h \rangle \]

\[ \langle \delta V^2 \rangle = (2\pi \bar{r} \bar{h})^2 \langle \delta r^2 \rangle + (2\pi \bar{r} \bar{h}) \langle \delta h^2 \rangle \]

\[ = (2\frac{\bar{V}}{\bar{r}})^2 \langle \delta r^2 \rangle + (\frac{\bar{V}}{\bar{h}})^2 \langle \delta h^2 \rangle \]

\[ = 0 \quad \text{usually} \]
\[ \frac{\sigma_v^2}{V^2} = \left( 2 \frac{\sigma_r}{r} \right)^2 + \left( \frac{\sigma_n}{n} \right)^2 \]

fractional errors in quadrature

\[ z = x + y \]

\[ \sigma_z^2 = \sigma_x^2 + \sigma_y^2 \]