1. Root-Mean-Squared is

$$RMS \equiv \left(\sum_{n=0}^{\infty} (n - \langle n \rangle)^2 P(n) \right)^{1/2}$$

$$\sum_{n=0}^{\infty} (n - \langle n \rangle)^2 P(n) = \sum_{n=0}^{\infty} n^2 P(n) - 2\langle n \rangle \overbrace{\sum_{n=0}^{\infty} n P(n)}^{\langle n \rangle} - \langle n \rangle^2 \underbrace{\sum_{n=0}^{\infty} P(n)}_{1}$$

$$= \sum_{n=0}^{\infty} n^2 P(n) - \langle n \rangle^2$$

$$\sum_{n=0}^{\infty} n^2 P(n) = \sum_{n=0}^{\infty} n^2 \frac{\langle n \rangle^n}{n!} e^{-\langle n \rangle} = \sum_{n=1}^{\infty} n \frac{\langle n \rangle^n}{(n-1)!} e^{-\langle n \rangle}$$

$$= \langle n \rangle^2 \underbrace{\sum_{n=2}^{\infty} \cancel{(n-1)} \frac{\langle n \rangle^{n-2}}{\cancel{(n-1)!}\,(n-2)!} e^{-\langle n \rangle}}_{=1} + \langle n \rangle \underbrace{\sum_{n=1}^{\infty} \frac{\langle n \rangle^{n-1}}{(n-1)!} e^{-\langle n \rangle}}_{=1}$$

$$\sum_{n=0}^{\infty} n^2 P(n) = \langle n \rangle^2 + \langle n \rangle \qquad \text{so}$$

$$\sum_{n=0}^{\infty} (n - \langle n \rangle)^2 P(n) = \sum_{n=0}^{\infty} n^2 P(n) - \langle n \rangle^2$$

$$= \cancel{\langle n \rangle^2} + \langle n \rangle - \cancel{\langle n \rangle^2}$$

$$= \langle n \rangle$$

so, the

$$\boxed{RMS = \sqrt{\langle n \rangle}}$$

2. $1 \text{ day} = 24 \cdot 60 \cdot 60 = 8.64 \cdot 10^{4}\,s \equiv \tau$

$$\langle n \rangle = 10^{34} \frac{1}{\cancel{cm^2}\,\cancel{s}} \cdot 10^{-39}\,\cancel{cm^2} \cdot (\tau = 8.64 \cdot 10^{4}\,\cancel{s})$$

$$= 8.64 \cdot 10^{-1} \text{ events}$$

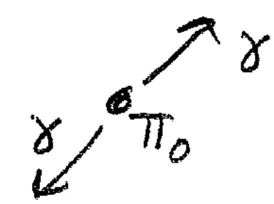
$$\boxed{\langle n \rangle = 0.864 \text{ events/day}}$$

3. G 6.6 Guess: $\mathcal{M} = \alpha\, m_{\pi^0} c$

Identical Photons: $S = \frac{1}{2}$

$$\Gamma = \frac{\frac{1}{2} \cdot c|\vec{p}|}{8\pi \hbar\, \cancel{m_{\pi^0}^2 c^2}} \alpha^2 \cancel{m_{\pi^0}^2 c^2} \qquad \left(\begin{matrix}6.35 \\ p. 208\end{matrix}\right)$$

but $c|\vec{p}| = \frac{1}{2} m_{\pi^0} c^2$



$$\Gamma = \frac{1}{32\pi} \frac{\alpha^2 m_{\pi^0} c^2}{\hbar}$$

$2E_\gamma = m_{\pi^0} c^2$

$|\vec{p}_\gamma| = \frac{1}{c} E_\gamma = \frac{1}{2} m_{\pi^0} c$

$$\tau = \frac{1}{\Gamma} = \frac{32\pi}{\alpha^2} \frac{\hbar c}{m_{\pi^0} c^2} \cdot \frac{1}{c}$$

$$= \frac{32\pi}{\left(\frac{1}{137}\right)^2} \cdot \frac{197 \text{ MeV-fm}}{135 \text{ MeV}} \frac{1}{3 \cdot 10^{23} \frac{fm}{s}}$$

$\boxed{\tau = 9.2 \cdot 10^{-18}\,s}$ ← estimate

$\boxed{= 8.4 \cdot 10^{-17}\,s}$ ← Measurement

within one order of magnitude

$\boxed{4.\ G\ 6.8}$

$$a + b \to a + b$$

$$m_b c^2 \gg E_a \quad \text{(initial)}.$$

Note: b is non-relativistic, so..

$$|\vec{p}_b'| = m_b |\vec{\beta}_b'| c$$

$$T_b' = E_b' - m_b c^2 = \frac{1}{2} m_b |\vec{\beta}_b'|^2 c^2$$

Since $|\vec{p}_b'| \propto |\vec{\beta}_b'|$, but $T_b' \propto |\vec{\beta}_b'|^2$, one can neglect energy of b in final state (one order higher than momentum in $|\vec{\beta}_b'|$).

Still, 6.38 p. 209 is

$$\sigma = \frac{S\hbar^2}{4\sqrt{(p_1 \cdot p_2)^2 - (m_1 m_2 c^2)^2}} \int |\mathcal{M}|^2 (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4)$$

$$\times \prod_{j=3}^{n} \frac{1}{2\sqrt{|\vec{p}_j|^2 + m_j^2 c^2}} \frac{d^3 \vec{p}_j}{(2\pi)^3}$$

$$E_1 \equiv E_a \quad E_2 = m_b c^2 \quad E_3 = E_a' \quad E_4 = m_b c^2$$

$$\vec{p}_1 \equiv \vec{p}_a \quad \vec{p}_2 = 0 \quad \vec{p}_3 = \vec{p}_a' \quad \vec{p}_4 = \vec{p}_b'$$

$$p_1 \cdot p_2 = \frac{E_1 E_2}{c^2} - \vec{p}_1 \cdot \vec{p}_2 = \frac{E_a \cdot m_b c^2}{c^2} - \vec{p}_a \cdot \vec{0}$$

$$= E_a m_b$$

$$\sqrt{(p_1 \cdot p_2)^2 - (m_1 m_2 c^2)^2} = \sqrt{E_a^2 m_b^2 - m_a^2 c^4 m_b^2} = m_b c |\vec{p}_a|$$

$$\delta^4(p_1+p_2-p_3-p_4)$$

$$= \underbrace{\delta\left(\frac{E_a}{c} + \cancel{m_b c} - \frac{E_a'}{c} - \cancel{m_b c}\right)}_{\text{will keep magnitude}} \delta^3(\vec{p}_a - \vec{p}_a{}' - \vec{p}_b{}')$$

$$|\vec{p}_a{}'| = |\vec{p}_a|$$

do $d^3\vec{p}_3{}'$ integral

$$\sigma = \frac{S\hbar^2}{64\pi^2 m_b c|\vec{p}_a|}\int |\mathcal{M}|^2 \underbrace{\frac{\delta\left(\frac{E_a}{c}-\frac{E_a'}{c}\right)\delta^3(\vec{p}_a-\vec{p}_a{}'-\vec{p}_b{}')}{\sqrt{|\vec{p}_a{}'|^2+m_a^2c^2}\cdot m_b c}\, d^3\vec{p}_a{}' d^3\vec{p}_b{}'}_{\text{no dependence on } \vec{p}_b{}'!}$$

$$= \frac{S\hbar^2}{64\pi^2(m_b c)^2|\vec{p}_a|}\int |\mathcal{M}|^2 \frac{\delta\left(\frac{1}{c}(E_a-E_a')\right)|\vec{p}_a{}'|^2\, d|\vec{p}_a{}'|\, d\Omega}{\sqrt{|\vec{p}_a{}'|^2+(m_a c)^2}}$$

$$\frac{1}{c}E_a' = \sqrt{|\vec{p}_a{}'|^2+(m_a c)^2}$$

$$\frac{1}{c}dE_a' = \frac{\frac{1}{2}2|\vec{p}_a{}'|}{\sqrt{|\vec{p}_a{}'|^2+(m_a c)^2}}\, d|\vec{p}_a{}'|$$

so

$$\frac{d\sigma}{d\Omega} = \frac{S\hbar^2}{(8\pi m_b c)^2|\vec{p}_a|}\int |\mathcal{M}|^2\,\delta\left(\frac{1}{c}(E_a-E_a')\right)|\vec{p}_a{}'|\frac{dE_a'}{c}$$

$$= \frac{S\hbar^2}{(8\pi m_b c)^2\cancel{|\vec{p}_a|}}\cancel{|\vec{p}_a|}\,|\mathcal{M}|^2 = \underline{\underline{\left(\frac{\hbar}{8\pi m_b c}\right)^2|\mathcal{M}|^2}}$$

$S=1 \quad a \neq b$

$\boxed{5.\ G\ 6.9}$ Last problem.

$$\sqrt{(p_1 \cdot p_2)^2 - (m_1 m_2 c^2)^2} = m_2 |\vec{p}_1| c$$

plugging in just before 6.42 p. 209

$$\sigma = \frac{S\hbar^2}{64\pi^2 m_2 |\vec{p}_1| c} \int |\mathcal{M}|^2 \frac{\delta^4(p_1+p_2-p_3-p_4)}{\sqrt{|\vec{p}_3|^2+m_3^2c^2}\sqrt{|\vec{p}_4|^2+m_4^2c^2}}\, d^3\vec{p}_3\, d^3\vec{p}_4$$

$$\delta^4(p_1+p_2-p_3-p_4) = \delta\left(\frac{E_1}{c} + m_2 c - \frac{E_3}{c} - \frac{E_4}{c}\right)\delta^3(\vec{p}_1 - \vec{p}_3 - \vec{p}_4)$$

particles 3+4 massless: $\frac{E_1}{c} = |\vec{p}_1|$ $\frac{E_3}{c} = |\vec{p}_3|$

natural to do integral over $d^3\vec{p}_4$, which then allows replacement $\vec{p}_4 = \vec{p}_1 - \vec{p}_3$

also $\frac{E_4}{c} = |\vec{p}_4| = |\vec{p}_1 - \vec{p}_3|$

For remaining integral over $d^3p_3 = |\vec{p}_3|^2 d|\vec{p}_3| d\Omega$, we get.. (now $d\Omega = \sin\theta\, d\theta\, d\phi$, $(\phi, \theta) \rightarrow$ describe #3's direction.)

$$\frac{d\sigma}{d\Omega} = \frac{S\hbar^2}{64\pi^2 m_2 |\vec{p}_1| c} \int \frac{|\mathcal{M}|^2 \delta(|\vec{p}_1| + m_2 c - |\vec{p}_3| - |\vec{p}_1 - \vec{p}_3|)\, |\vec{p}_3|^{\cancel{2}}\, d|\vec{p}_3|}{\cancel{|\vec{p}_3|}\, |\vec{p}_1 - \vec{p}_3|}$$

Conceptually, the δ-function fixes ~~the~~ magnitude of $|\vec{p}_3|$. What is complicated is how it does so...

$$\delta(|\vec{p}_1| + m_2 c - |\vec{p}_3| - |\vec{p}_1 - \vec{p}_3|) = \delta(f(|\vec{p}_3|) - |\vec{p}_1| - m_2 c)$$

$$f(|\vec{p}_3|) = |\vec{p}_3| + |\vec{p}_1 - \vec{p}_3|$$
$$= |\vec{p}_3| + \sqrt{|\vec{p}_1|^2 + |\vec{p}_3|^2 - 2|\vec{p}_1|\cdot|\vec{p}_3|\cos\theta}$$

$$df = \left(1 + \frac{|\vec{p}_3| - |\vec{p}_1|\cos\theta}{\sqrt{|\vec{p}_1|^2 + |\vec{p}_3|^2 - 2|\vec{p}_1||\vec{p}_3|\cos\theta}}\right) d|\vec{p}_3|$$

$$= \frac{\sqrt{|\vec{p}_1|^2 + |\vec{p}_3|^2 - 2|\vec{p}_1||\vec{p}_3|\cos\theta} + |\vec{p}_3| - |\vec{p}_1|\cos\theta}{\sqrt{|\vec{p}_1|^2 + |\vec{p}_3|^2 - 2|\vec{p}_1|\cdot|\vec{p}_3|\cos\theta}}\, d|\vec{p}_3|$$

$$df = \frac{|\vec{p}_1 - \vec{p}_3| + |\vec{p}_3| - |\vec{p}_1|\cos\theta}{|\vec{p}_1 - \vec{p}_3|}\, d|\vec{p}_3|$$

or
$$\frac{df}{|\vec{p}_1 - \vec{p}_3| + |\vec{p}_3| - |\vec{p}_1|\cos\theta} = \frac{d|\vec{p}_3|}{|\vec{p}_1 - \vec{p}_3|}$$

so, we get

$$\frac{d\sigma}{d\Omega} = \frac{S\hbar^2}{64\pi^2 m_2 |\vec{p}_1| c} \int \frac{|\mathcal{M}|^2 \delta(|\vec{p}_1| + m_2 c \overbrace{- |\vec{p}_3| - |\vec{p}_1 - \vec{p}_3|}^{-f(|\vec{p}_3|)})\, |\vec{p}_3|\, df}{\underbrace{|\vec{p}_1 - \vec{p}_3| + |\vec{p}_3|}_{f(|\vec{p}_3|)} - |\vec{p}_1|\cos\theta}$$

$f(|\vec{p}_3|)$ using δ function, this becomes $|\vec{p}_1| + m_2 c$. Factor in c, $c|\vec{p}_1| = E_1$, get $m_2 c^2$, $c|\vec{p}_1|\cos\theta$

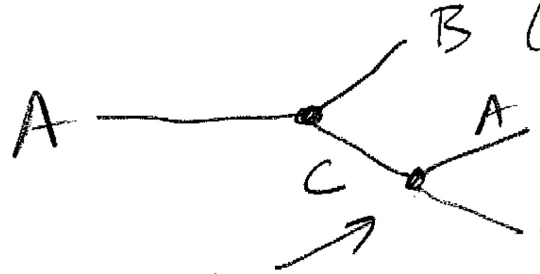
finally

$$\boxed{\frac{d\sigma}{d\Omega} = \left(\frac{\hbar}{8\pi}\right)^2 \frac{S|\mathcal{M}|^2 |\vec{p}_3|}{m_2|\vec{p}_1|\left(E_1 + m_2 c^2 - |\vec{p}_1| c\cos\theta\right)}}$$

6. G 6.11

(a) $A \to B + B$ Try drawing Feynman Diagram

$\to$ ONLY ABC Vertex



(got one B, leave it!

$\leftarrow$ can we get rid of A? NO.

got B!

Try to turn into B

$\to$ What you do get is $A \to A + B + B$

Could link A to upper B, but you get C.

Could "decay" the A to B + C, but

then $A \to B + B + B + C$

In the end, $\boxed{A \to B + B \text{ not possible}}$

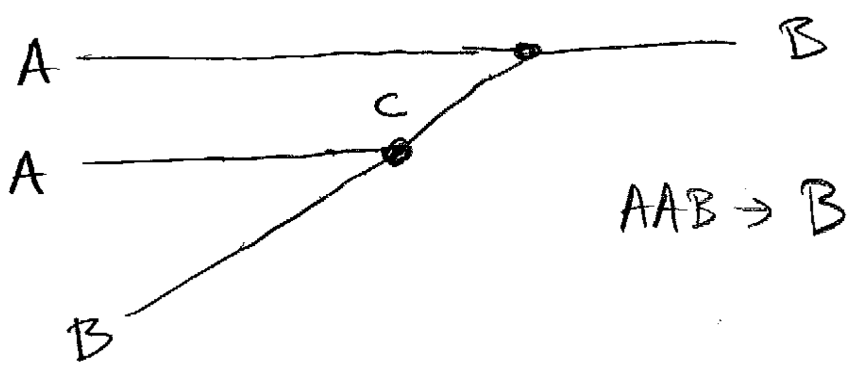
(b) Using part (a) as a guide, the ideas are:

(1) The ABC vertex allows cancellation of 1A line, 1B line, 1C line. That is, it is always possible to link those lines with a vertex.

Algebraically, let $x = \text{minimum}(n_A, n_B, n_C)$

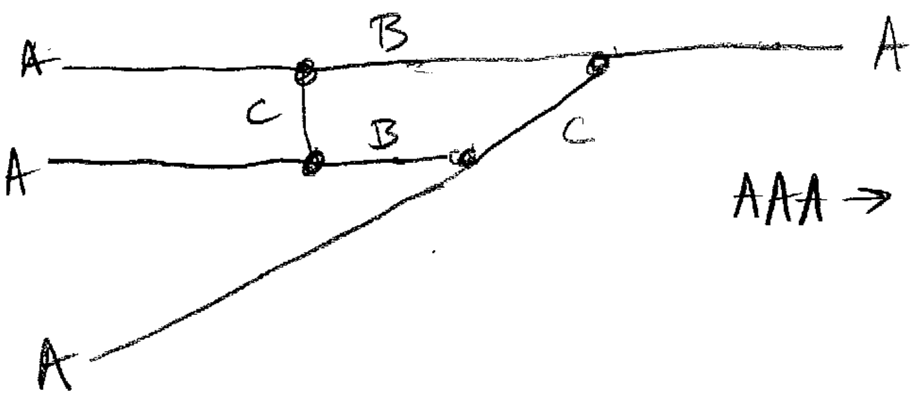
define $n_A' = n_A - x \quad n_B' = n_B - x \quad n_C' = n_C - x$

(2) Then, if n_A', n_B', + n_C' are all either 0 or <u>even</u>, the reaction is allowed. That is because a <u>pair</u> of A's (or B's, or C's) can always be "absorbed" by any line..



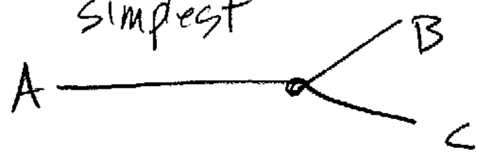
$AAB \to B$ ($AAC \to C$ works similarly).

or



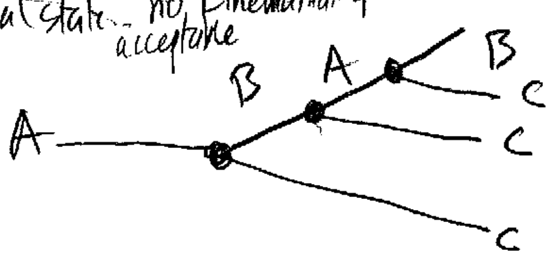
$AAA \to A$

(c) $A \to B + C$ simplest



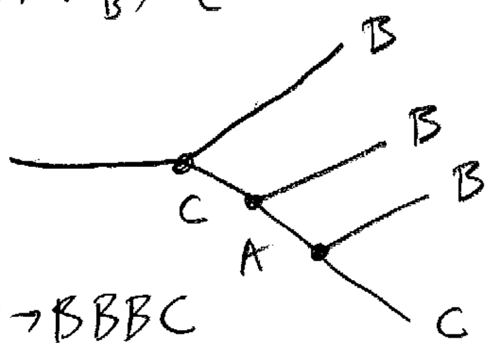
$O(g)$

$O(g^2)$ all have A in final state.. no kinematically acceptable assuming $m_A \gg m_B, m_C$



$A \to BCCC$

or $O(g^3)$



$A \to BBBC$

7. $\boxed{\text{G 6.14}}$ $AA \to BB$, $m_B = m_C = 0$

$m_A \neq 0$

Can use results from:

(A) # 6.9, done earlier.

(B) 6.32 → equation 6.55

$$6.55:\quad \mathcal{M} = g^2 \left[\frac{1}{(p_4 - p_2)^2 - 0} + \frac{1}{(p_3 - p_2)^2 - 0} \right]$$

$m_C = 0$

$$= g^2 \underbrace{\left[\frac{1}{(p_4 - p_2)^2} + \frac{1}{(p_3 - p_2)^2} \right]}_{\text{can evaluate in any frame}}$$

In <u>lab</u>, 6.9 says...

$$\frac{d\sigma}{d\Omega} = \left(\frac{\hbar}{8\pi}\right)^2 \frac{\cdot \frac{1}{2} \cdot |\vec{p}_3|}{m_A |\vec{p}| (E + m_A c^2 - |\vec{p}| c \cos\theta)} |\mathcal{M}|^2$$

Wow.. turned out to be quite involved! First, get $|\vec{p}_3|$ as a function of θ...

$$p_1 + p_2 = p_3 + p_4$$

$$p_1^2 + \cancel{p_2^2} + 2 p_1 \cdot p_2 = \cancel{p_3^2} + \cancel{p_4^2} + 2 p_3 \cdot p_4$$

$$2 m_A^2 c^2 + 2 E m_A = 2 p_3 \cdot p_4$$

$$p_3 \cdot p_4 = m_A^2 c^2 + m_A E = m_A (m_A c^2 + E)$$

<u>also</u> $$p_3 \cdot (p_3 + p_4) = \underbrace{\cancel{p_3^2}}_{0} + p_3\, p_4 = p_3 (p_1 + p_2)$$

$$p_3 \cdot p_4 = \underbrace{(p_3 \cdot p_1)}_{\frac{E E_3}{c^2} - \frac{E_3}{c}|\vec{p}|\cos\theta} + \underbrace{(p_3\, p_2)}_{+ E_3 \cdot m_A}$$

$$= \frac{E_3}{c^2}\left(m_A c^2 + E - c|\vec{p}|\cos\theta \right)$$

so $$E_3 = c|\vec{p}_3| = \frac{m_A c^2 (E + m_A c^2)}{E + m_A c^2 - c|\vec{p}|\cos\theta}$$

so,

$$\frac{d\sigma}{d\Omega} = \frac{1}{2}\left(\frac{\hbar}{8\pi}\right)^2 \frac{c\,(E + m_A c^2)}{|\vec{p}|\left(E + m_A c^2 - c|\vec{p}|\cos\theta\right)^2} |\mathcal{M}|^2$$

<u>next</u>.

$$p_1 + p_2 = p_3 + p_4$$

$$(p_4 - p_2) = (p_1 - p_3)$$

$$(p_4 - p_2)^2 = p_1^2 + \underbrace{\cancel{p_3^2}}_{0} - 2 p_1 p_3 = m_A^2 c^2 - 2 \frac{E_3}{c^2}\left[E - c|\vec{p}|\cos\theta\right]$$

$$(p_3 - p_2)^2 = \cancel{p_3^2} + \cancel{p_2^2} - 2 p_2 p_3$$

$$= m_A^2 c^2 - 2 m_A E_3$$

$$\frac{1}{(p_4-p_2)^2}+\frac{1}{(p_3-p_2)^2}$$

$$=\frac{1}{m_A^2c^2-\frac{2E_3}{c^2}\left[E-c|\vec{p}|\cos\theta\right]}+\frac{1}{m_A^2c^2-2m_AE_3}$$

$$=\frac{2m_A^2c^2-\frac{2E_3}{c^2}\left[E+m_Ac^2-c|\vec{p}|\cos\theta\right]}{\left(m_A^2c^2-\frac{2E_3}{c^2}\left[E-c|\vec{p}|\cos\theta\right]\right)\left(m_A^2c^2-2m_AE_3\right)}$$

but $\underline{E_3\cdot\left(E+m_Ac^2-c|\vec{p}|\cos\theta\right)=m_Ac^2\left(E+m_Ac^2\right)}$

numerator $\quad 2m_A^2c^2-\frac{2}{\cancel{c^2}}\,m_A\cancel{c^2}\left(E+m_Ac^2\right)=-2m_AE$

$\boxed{\text{denominator}}$

$$\left(m_A^2c^2\right)^2\underbrace{-\frac{2E_3}{c^2}\left(m_A^2c^2\left(E+m_Ac^2-c|\vec{p}|\cos\theta\right)\right)}+\frac{4E_3^2}{c^2}m_A\left[E-c|\vec{p}|\cos\theta\right]$$

$$-2m_A^3c^2\left(E+m_Ac^2\right)+\frac{4E_3^2}{c^2}m_A\left[E-c|\vec{p}|\cos\theta\right]$$

$$-(m_Ac)^4-2m_A^3c^2E+\frac{4E_3^2}{c^2}m_A\left[E-c|\vec{p}|\cos\theta\right]$$

... will post completion after final..

H.N.