Physics 125 Fifth Problem Set

Harry Nelson
TA: Ted Brookings

due Monday, May 13, 2001

This covers lecture material. It is in your interest to understand the first two problems before the midterm on Wed., May 8.

1. Particle #1 has mass m_1 and charge q; particle #2 has mass m_2 and charge $-q$. The two particles are in a classical bound state, where each follows a circular trajectory about their center of mass. The period of their orbit is T, and the distance between them is r.

   (a) The *net* charge of the system is 0. Under what conditions on m_1 and m_2 will the magnetic moment of the system be 0?

   (b) What is the general expression for the magnetic moment of the system? What is the value of the fudge-factor (the gyromagnetic ratio) g?

2. You send a beam of spin 1/2 atoms through three Stern-Gerlach measurements:

   (a) In the first, $s_z = +\hbar/2$ is accepted, and $s_z = -\hbar/2$ is rejected.

   (b) In the second, the direction of the $\vec{B}$ field in the magnet is rotated about y by an angle α from the z direction toward the x direction; call the direction of the new axis $\hat{n}$, and the component of spin along it s_n. In this second measurement $s_n = +\hbar/2$ is accepted and $s_n = -\hbar/2$ is rejected.

   (c) In the third, the direction of the $\vec{B}$ field is again in the z direction, but now $s_z = -\hbar/2$ is accepted, and $s_z = +\hbar/2$ is rejected.

   Find the intensity of the beam exiting the third measurement *relative* to the intensity of the beam exiting the first measurement. For what value of α is the relative intensity maximized?

3. This (lengthy) problem addresses the actual measurement of the z component of spin in a Stern-Gerlach magnet. The punch line will that the actual measurement of s_z causes decoherence of the spin components that are perpendicular to z, due to the diffraction of the atoms (which are waves too!) in the Stern-Gerlach magnet.

   First, imagine the spreading of the wavefunction in the z direction in the *absence* of any magnetic field in the Stern-Gerlach magnet; the situation is depicted in Fig. 1(a). In the figure, a particle with wavefunction ψ enters with only the y-component of 3-momentum p_y non-zero, from the left into the bore of a Stern-Gerlach magnet. The full-width-at-half-max of the wavefunction in z is initially Δz. Consequently, the wavefunction spreads due to diffraction, with spreading angle $\Delta\phi = \lambda/\Delta z$.

   (a) Express $\Delta\phi$ in terms of fundamental constants $2\pi\hbar$, and p_y, assuming that $|p_y| \gg |p_x|, |p_z|$.

   Second, imagine turning the field on, as depicted in Fig. 1(b). The existence of a non-zero $|\partial B_z/\partial z|$ gives spin-up and spin-down *impulses* in the z direction equal to $\pm FT$, where F is the magnitude of force on the atoms, and T is the time spent traversing the length of the magnet.

   (b) Evaluate the deflection angle ϕ in terms of g, e, m_e, $\hbar$, p_y, $\partial B_z/\partial z$, T, and any other quantity you think is relevant.

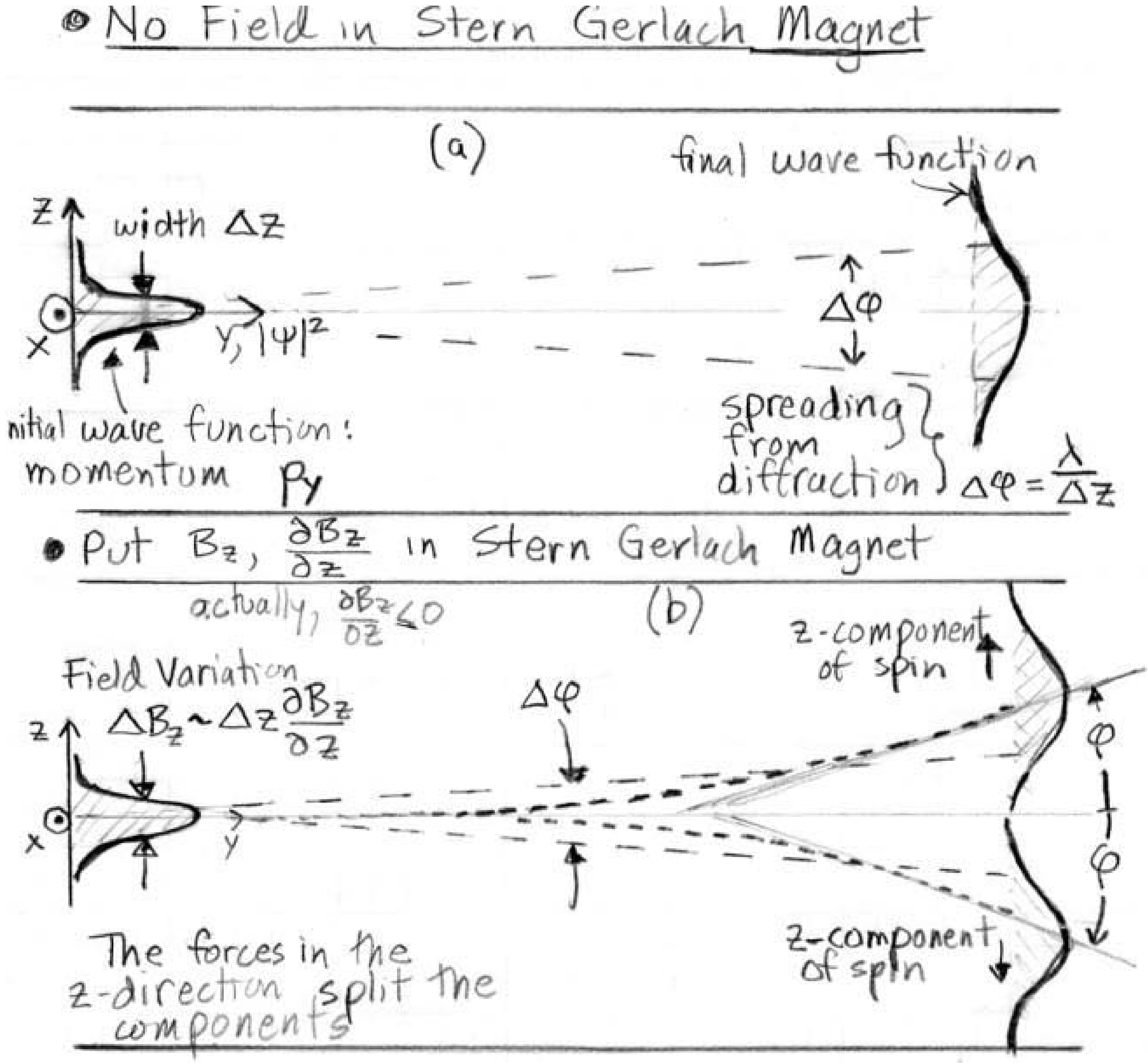

Figure 1: For Problem 3. In the upper figure (a), the spreading or diffraction of the wavefunction ψ in z is depicted, in absence of any magnetic field in the Stern-Gerlach magnet. In the lower figure (b), the effect of the a non-zero field derivative $\partial B_z/\partial z$ is depicted; the impulse transferred to the s_z-up and the s_z-down components must cause deflections of the beams sufficient to cause an angle between the two components that exceeds the spreading angle from wave diffraction.

(c) For the two beams to separate (eventually) so that they are distinguishable one must have $2\phi > \Delta\phi$. Use this relationship, but rearrange it, to place a lower bound on $\Delta\omega_p T$. Here $\Delta\omega_p$ is the *difference* in precession frequency between the parts of the wavefunction ψ at higher z and lower z, which are separated by Δz.

(d) To visualize the concept of spin coherence, imagine that at each point in space where the wave function ψ is non-zero, there can be a distinct value of each of the three components of spin. Focus, for simplicity, on just s_x. Just before entering the Stern-Gerlach magnet with non-zero field, suppose that the value of the x component of spin s_x is non-zero, and constant across the whole spatial extent of ψ. What does the answer to part (c) imply about the spatial distribution of s_x when the wavefunction *exits* that Stern-Gerlach magnet? Can you describe why the measurement of s_z wipes out the mean value, averaged across the whole wavefunction, of s_x?