Transitions

Decay $A \rightarrow B + C$

$$\pi^- \rightarrow \mu^- \bar{\nu}$$

$$\underset{m_\pi c^2}{139.6} \quad \underbrace{\underset{m_\mu c^2}{105.7}}_{\text{energy appears as momentum (kinetic energy) How much?}} \quad \approx 0 \quad \text{MeV}$$

"energy release"

not "elastic"

Scattering $A + B \rightarrow C + D$

elastic: $e^- + \mu^- \rightarrow e^- + \mu^-$ (no identity change)

inelastic: $\pi^- + p \rightarrow \Lambda + K^0$ (identities change)

(MeV) $\underbrace{139.7 + 938.3}_{\text{must input kinetic energy to make reaction to How much?}} < 1115.7 + 497.7 \leftarrow$ masses

A useful view of scattering:

$$A + B \Rightarrow X \xrightarrow{\text{decays}} C + D$$

if this is moving, some of A+B's energy is "wasted," because <u>kinetic</u> energy of X cannot be converted into mass of C+D

useful frame of reference:

$\vec{p}^* \Longrightarrow$ $A \xrightarrow{\vec{p}_A} X \xleftarrow{-\vec{p}_A = \vec{p}_B} B$ $\leftarrow$ A+B system "at rest"

born at rest

$\vec{q}^* \longrightarrow$ $C \xleftarrow{\vec{p}_C} X \xrightarrow{-\vec{p}_C = \vec{p}_D} D$

$|\vec{p}^*| > |\vec{q}^*|$ $m_A + m_B < m_C + m_D$

$|\vec{p}^*| < |\vec{q}^*|$ $m_A + m_B > m_C + m_D$

Example #1 : $\pi^- \to \mu^- \bar{\nu}_\mu$ ← $m_{\nu_\mu} \simeq 0$ (easy)

$m_\pi c^2 = 139.6$ MeV $\quad m_\mu c^2 = 105.7$ MeV

Example #2 $\quad D^0 \to K^- \pi^+$ ← $m_\pi c^2 = 139.6$ MeV

$m_{D^0} c^2 = 1865$ MeV $\quad m_K c^2 = 493.7$ MeV

$$E_A = m_A c^2 = \sqrt{(m_B c^2)^2 + c^2|\vec{p}^*|^2} + \sqrt{(m_c c^2)^2 + c^2|\vec{p}^*|^2}$$

work in rest frame of A $\quad \vec{p}_A = \vec{p}^* \qquad \vec{p}_C = -\vec{p}^*$

simplifies when:

(A) $m_c = 0$ ($\pi^- \to \mu^- \bar{\nu}_\mu$) $\qquad (m_A c^2 - c|\vec{p}^*|)^2 = (m_B c^2)^2 + |\vec{p}^*|^2 c^2$

$$m_A^2 c^4 - 2 m_A c^2 c|\vec{p}^*| + \cancel{c^2|\vec{p}^*|^2} = (m_B c^2)^2 + \cancel{|\vec{p}^*|^2 c^2}$$

$$\boxed{|\vec{p}^*| = \frac{m_A^2 - m_B^2}{2 m_A} c}$$

Same when $m_B = 0$

(B) $m_C = m_B$

$$\frac{m_A c^2}{2} = \sqrt{(m_B c^2)^2 + c^2|\vec{p}^*|^2}$$

$$\boxed{|\vec{p}^*| = \sqrt{(\tfrac{1}{2} m_A)^2 - m_B^2} \cdot c}$$

General Case

- use "invariant" 4-dot product
- go to a simple frame
- get E_B^*, E_c^* first, then $|\vec{p}^*|^2 = E_A^{*2} - m_A^2 = E_B^{*2} - m_B^2$

$$p_A = p_B + p_C \quad \Leftarrow \text{ a 4-vector equation.}$$

$p_B \cdot \qquad\qquad p_C \cdot$

$$p_B \cdot p_A = \underbrace{p_B^2}_{m_B^2 c^2} + p_B \cdot p_C \qquad p_C \cdot p_A = p_C \cdot p_B + \underbrace{p_C^2}_{m_c^2 c^2}$$

In the rest frame of the A, $p_B \cdot p_A$ and $p_C \cdot p_A$ are very simple:

$$p_A = (m_A c, \vec{0}) \quad p_B = \left(\frac{E_B^*}{c}, \vec{p}^*\right) \quad p_C = \left(\frac{E_C^*}{c}, -\vec{p}^*\right)$$

$$p_B \cdot p_A = \left(\frac{E_B^*}{c}\right)(m_A c) - \vec{p}^* \cdot \vec{0} = m_A E_B^*$$

$$p_C \cdot p_A = \left(\frac{E_C^*}{c}\right)(m_A c) - -\vec{p}^* \cdot \vec{0} = m_A E_C^*$$

How about $p_B \cdot p_C = p_C \cdot p_B$? "square" the original equation:

$$p_A^2 = (p_B + p_C)^2 = p_B^2 + p_C^2 + 2 p_B \cdot p_C$$

$$m_A^2 c^2 = m_B^2 c^2 + m_C^2 c^2 + 2 p_B \cdot p_C$$

$$p_B \cdot p_C = \frac{1}{2}(m_A^2 - m_B^2 - m_C^2) c^2$$

plugging in:

$$m_A E_B^* = m_B^2 c^2 + \frac{1}{2}(m_A^2 - m_B^2 - m_C^2) c^2 \qquad m_A E_C^* = m_C^2 c^2 + \frac{1}{2}(m_A^2 - m_B^2 - m_C^2) c^2$$

$$\boxed{E_B^* = \frac{m_A^2 + m_B^2 - m_C^2}{2 m_A} c^2 \qquad E_C^* = \frac{m_A^2 - m_B^2 + m_C^2}{2 m_A} c^2}$$

$$|\vec{p}^*|^2 = \left(\frac{E_B^*}{c}\right)^2 - m_B^2 c^2 = \left(\frac{E_C^*}{c}\right)^2 - m_C^2 c^2$$

$$= \frac{(m_A^2 + m_B^2 - m_C^2)^2 - 4 m_A^2 m_B^2}{4 m_A^2} c^2 = \frac{(m_A^2 - m_B^2 + m_C^2)^2 - 4 m_A^2 m_C^2}{4 m_A^2} c^2$$

$$\boxed{|\vec{p}^*|^2 = \frac{m_A^4 + m_B^4 + m_C^4 - 2 m_A^2 m_B^2 - 2 m_A^2 m_C^2 - 2 m_B^2 m_C^2}{4 m_A^2} c^2}$$

$$\lambda(m_A^2, m_B^2, m_C^2) = m_A^4 + m_B^4 + m_C^4 - 2 m_A^2 m_B^2 - 2 m_A^2 m_C^2 - 2 m_B^2 m_C^2$$

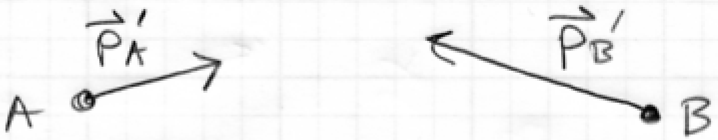
so, $$|\vec{p}^*| = \frac{\sqrt{\lambda(m_A^2, m_B^2, m_C^2)}}{2 m_A} c$$

Scattering

A scatters off B

m_A $\qquad$ m_B

$\vec{p}_A'$ $\qquad$ $\vec{p}_B'$



what is the particle (X) of largest mass that one can make here? m_X

Conceptually, go to frame where $\vec{p}_A = -\vec{p}_B$

$\vec{p}^* =$

$$m_X c^2 = E_A^* + E_B^* = \sqrt{(m_A c^2)^2 + |c\vec{p}^*|^2} + \sqrt{(m_B c^2)^2 + |c\vec{p}^*|^2}$$

$$\vec{p}_X = 0 = \vec{p}_A + \vec{p}_B = \vec{p}^* - \vec{p}^* = 0$$

or $\quad p_X = p_A + p_B$

$$p_X^2 = p_X^\mu p_{X\mu} = (p_A + p_B)^2 \leftarrow \text{FRAME INDEPENDENT}$$

$$= p_A^2 + p_B^2 + 2 p_A \cdot p_B$$

$$s \equiv \boxed{m_X^2 c^2 = m_A^2 c^2 + m_B^2 c^2 + 2\left(\frac{E_A E_B}{c^2} - \vec{p}_A \cdot \vec{p}_B\right)}$$

can evaluate this in any frame

Reasoning with the center of momentum frame leads to a formula that is more general.. applicable in any frame.

Example: $\pi^- + p \rightarrow \Lambda + K^0$

$\bar{u}d \quad uud \quad sdu \quad \bar{s}d$

electric charge		-1	$+1$	$=$	0	$+\ 0$		
net quarks (baryon #)		0	$+\ 3$	$=$	3	$+\ 0$		
		0	1		1	0		
net quark flavor	u	-1	$+\ 2$	$=$	1	$+\ 0$	✓	not necessarily weak ⇒ strong.
	d	1	$+\ 1$	$=$	1	$+\ 1$	✓	
	s	0	$+\ 0$	$=$	1	$-\ 1$	✓	

m: $139.6 + 938.3 < 1115.7 + 497.7$

need energy

π^- $\bullet\!\longrightarrow$ p $\circ$ (not center of mass)

what is minimum momentum necessary to get above threshold for $\Lambda + K^0$?

want $\underbrace{(p_\pi + p_p)^2}_{\text{4-vectors}} = \underbrace{m_X^2 c^2}_{\text{hypothetical intermediate}} > \underbrace{(m_\Lambda + m_{K^0})^2 c^2}_{\text{minimum mass for intermediate}}$

evaluate in lab ↓

$= p_\pi^2 + p_p^2 + 2 p_\pi \cdot p_p = m_\pi^2 c^2 + m_p^2 c^2 + 2\left(\frac{E_\pi}{c} \times m_p c - \vec{0}\right)$

$$E_\pi > \frac{(m_\Lambda + m_{K^0})^2 c^4 - m_\pi^2 c^4 - m_p^2 c^4}{2 m_p c^2} = 907.6 \text{ MeV}$$

$E_\pi = (m_\pi^2 c^4 + |\vec{p}_\pi|^2 c^2)^{1/2}$, $\boxed{c|\vec{p}_\pi| = \sqrt{E_\pi^2 - m_\pi^2 c^4} = 896.8 \text{ MeV}}$

↳ larger than center-of-mass momentum necessary: like decay of $m_\Lambda + m_{K^0}$ particle <u>in reverse</u>

π $\bullet\!\longrightarrow \quad \longleftarrow\!\bullet$ p

$\vec{p}^* \quad -\vec{p}^*$

$$c|\vec{p}^*| = \frac{\sqrt{\lambda((m_\Lambda + m_K)^2, m_\pi^2, m_p^2)}}{2(m_\Lambda + m_K)}$$

$\boxed{c|\vec{p}^*| = 521.5 \text{ MeV}}$