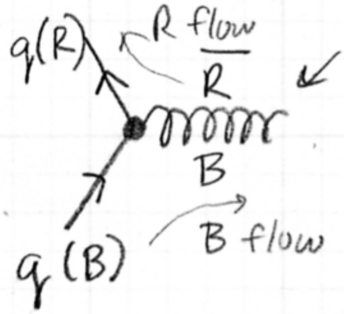
Strong Interaction

- Only quarks + gluons participate
- now each gluon has a color + an anticolor
- Basic vertex:

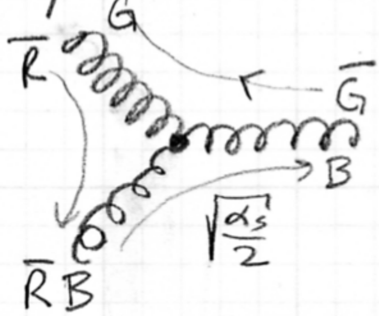


← gluon like a spring

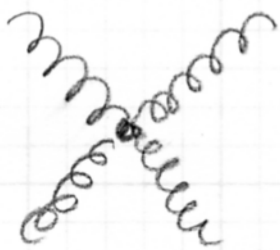
coupling actually $\sqrt{\frac{\alpha_s}{2}}$
same for all quarks

- What about "color neutral" case

Gluons actually couple to themselves, since they have color charge (and anticharge)



also



α_s generally $\gg 1$, so, the "lowest order" diagram generally greatly underestimates the amplitude. However, the lowest order diagram still gives some guidance.

The first process: where does the "ninth" gluon go? Consider:

B$\overline{B}$ is not an eigenstate

$$i\hbar \frac{\partial}{\partial t} |G\overline{G}\rangle = A |B\overline{B}\rangle$$

time change of G$\overline{G}$

amplitude: not just $\propto \left(\sqrt{\frac{\alpha_s}{2}}\right)^2$

"Color Symmetry" → amplitude is the same for all color transmutations.

$\langle R\bar{R}|\psi\rangle = g_1 \quad \langle G\bar{G}|\psi\rangle = g_2 \quad \langle B\bar{B}|\psi\rangle = g_3$

then $i\hbar\frac{\partial}{\partial t}\begin{pmatrix} g_1 \\ g_2 \\ g_3 \end{pmatrix} = A\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} g_1 \\ g_2 \\ g_3 \end{pmatrix}$

what are the eigenstates? The Hamiltonian is so simple, with so much symmetry, I'll just "guess"

$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{pmatrix} = 3\begin{pmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{pmatrix}$ so $\frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ is eigenvector, eigenvalue $3A$

$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \\ 0 \end{pmatrix} = 0\begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \\ 0 \end{pmatrix}$ so $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$ is eigenvector, eigenvalue 0

$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ -2/\sqrt{2} \end{pmatrix} = 0\begin{pmatrix} 1/\sqrt{6} \\ 1/\sqrt{6} \\ -2/\sqrt{6} \end{pmatrix}$ so $\frac{1}{\sqrt{6}}\begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$ is eigenvector, eigenvalue 0

The last two are not unique, because of the degeneracy; those are the "conventional" choices

For $\frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, the eigenvalue of energy is $3A$, and $3A \gg$ current energies. So, the "ninth gluon" described by:

$$|G_9\rangle = \frac{1}{\sqrt{2}}\left(|R\bar{R}\rangle + |G\bar{G}\rangle + |B\bar{B}\rangle\right) \quad \text{is so}$$

massive it does not participate in low-energy interactions.

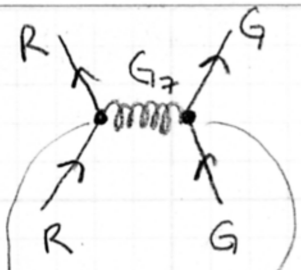
The other two,

$$|G_7\rangle = \frac{1}{\sqrt{2}}\left(|R\bar{R}\rangle - |G\bar{G}\rangle\right) \text{ and } \frac{1}{\sqrt{6}}\left(|R\bar{R}\rangle + |G\bar{G}\rangle - 2|B\bar{B}\rangle\right) = |G_8\rangle$$

have 0 mass, just like $|R\bar{G}\rangle$, $|R\bar{B}\rangle$, etc

How 2 Quarks Can be Attracted

2 electrons <u>repel</u> via electromagnetic interaction. but 2 quarks of same type can <u>attract</u> by taking advantage of the degree of freedom provided by color. The attraction occurs when the two quarks have different colors.



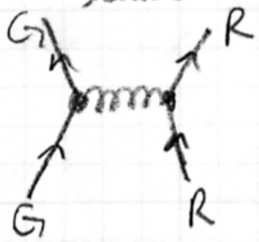
$\sqrt{\frac{\alpha_s}{2}}\cdot\frac{1}{\sqrt{2}}\times\sqrt{\frac{\alpha_s}{2}}\times -\frac{1}{\sqrt{2}}$ $\quad$ $\sqrt{\frac{\alpha_s}{2}}\cdot\frac{1}{\sqrt{6}}\cdot\sqrt{\frac{\alpha_s}{2}}\cdot\frac{1}{\sqrt{6}}$ $\quad$ $\frac{\alpha_s}{2}$

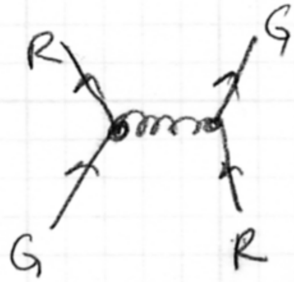
whoops, $q(R)q(G)$ not an eigenstate!!

$= -\frac{\alpha_s}{4}$ $\quad$ $= +\frac{\alpha_s}{12}$

same

$-\frac{1}{4}+\frac{1}{12} = -\frac{1}{6}$





$a_1 = \langle RG|\Psi\rangle \qquad a_2 = \langle GR|\Psi\rangle$

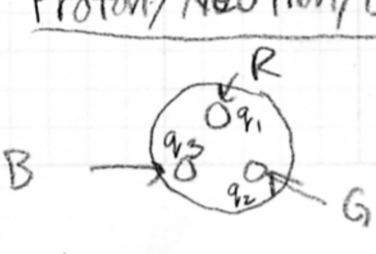
and $i\hbar\frac{\partial}{\partial t}\begin{pmatrix}a_1\\a_2\end{pmatrix} = \alpha_s\begin{pmatrix}-1/6 & 1/2\\ 1/2 & -1/6\end{pmatrix}$ ← very symmetric

eigenvectors by guessing: $\begin{pmatrix}-1/6 & 1/2\\ 1/2 & -1/6\end{pmatrix}\begin{pmatrix}1/\sqrt{2}\\1/\sqrt{2}\end{pmatrix} = \frac{1}{6}\begin{pmatrix}1/\sqrt{2}\\1/\sqrt{2}\end{pmatrix}$ eigenvalue $+\frac{\alpha_s}{6}$

$\begin{pmatrix}-1/6 & 1/2\\ 1/2 & -1/6\end{pmatrix}\begin{pmatrix}1/\sqrt{2}\\-1/\sqrt{2}\end{pmatrix} = -\frac{2}{3}\begin{pmatrix}1/\sqrt{2}\\-1/\sqrt{2}\end{pmatrix}$ eigenvalue $-\frac{2}{3}\alpha_s$

The punchline: $\frac{1}{\sqrt{2}}\left(|q(R)q(B)\rangle - |q(B)q(R)\rangle\right)$
"asymmetric"
has negative (attractive) energy, magnitude $-\frac{2}{3}\alpha_s$

Proton/Neutron/Baryon: $c_1 = R \quad c_2 = G \quad c_3 = B$



$|\text{Baryon}\rangle \propto \sum \varepsilon_{ijk}|q_1(c_i)q_2(c_j)q_3(c_k)\rangle$

$\varepsilon_{ijk} = 0$ if any 2 indices same

$\varepsilon_{123} = \varepsilon_{312} = \varepsilon_{231} = 1; \quad \varepsilon_{213} = \varepsilon_{321} = \varepsilon_{132} = -1$

So, if you exchange any pair of colors (or, separately, quarks) is swapped, the state comes back to itself, multiplied by -1 ... "totally asymmetric"!

$\Rightarrow$ "TOTALLY ATTRACTIVE" (you are held together)

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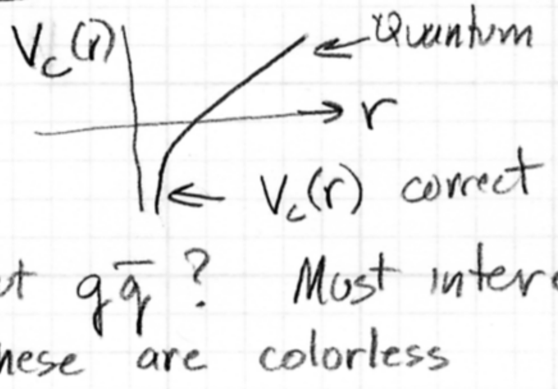
note: • spatial dependence of strong potential has been ignored

• at <u>short</u> distance, $V_c(r) = -\frac{2}{3}\frac{\alpha_s}{r}$ qq, "asymmetric"



• How about $q\bar{q}$? Most interesting when these are colorless

$-$ sign since "anti-color"

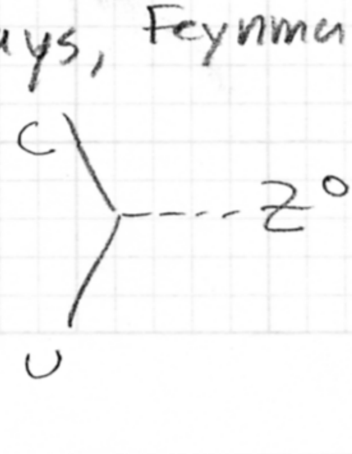
G_7: $+\frac{1}{\sqrt{2}}$ $+\frac{1}{\sqrt{2}}$ $\Big\}$ G_7 amplitude $-\frac{\alpha_s}{2} \times \frac{1}{2}$

<u>you</u> do G_8, + others ($R\bar{R} \to G\bar{G}$ etc)

ignore annihilation diagrams, only do "t-channel"

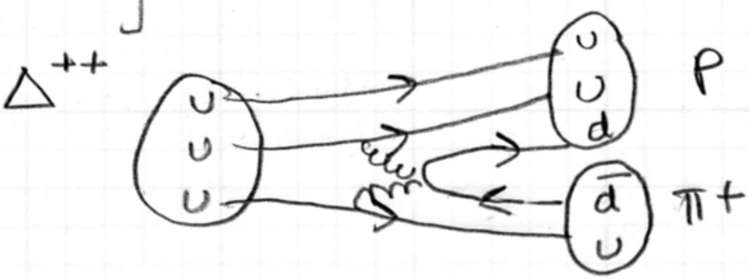
Other comments about decays, Feynman diagrams:

1) Z^0 like a photon; never

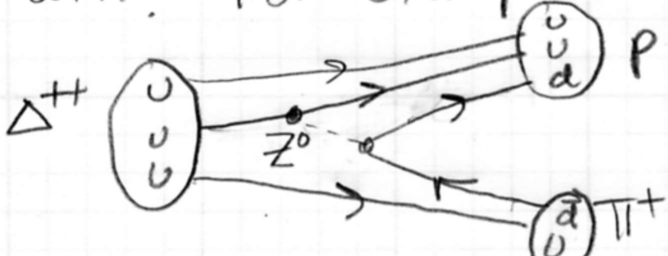


BIG MYSTERY

2) Strong interaction can drive decays.



- no quark changes flavor in strong decays
- quark-antiquark pair often "pops" from glue – usually $d\bar{d}$ or $u\bar{u}$; rarely $s\bar{s}$; $c\bar{c}$, $b\bar{b}$, $t\bar{t}$ too heavy, suppressed
- if a decay can go strong, it will! For example:

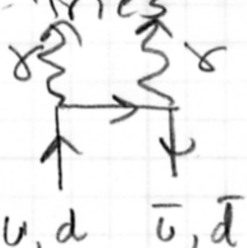


can go, but Branching ratio negligible.

3) Symmetry between u + d can complicate strong decays.. "isospin"

4) Sometimes strong can't go.. then, electromagnetic tries

$\pi^0 \to \gamma\gamma$



always γ's or e^+e^- in final state

5) when strong, em can't go, weak can (changing flavor of quarks).