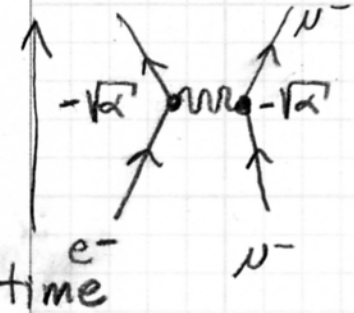
e^- interacting with μ^-

"lowest order" Feynman Diagram



"amplitude" $\propto$ product of charges or coupling constants

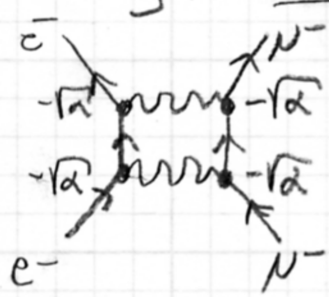
$$\propto (-\sqrt{\alpha})(-\sqrt{\alpha}) = +\alpha \quad \left\{\frac{e^2}{\hbar c}\right\}$$

what physics is proportional to the amplitude? Amplitude $\Longleftrightarrow$ Expectation Value of Hamiltonian

$\Longleftrightarrow$ energy due to interaction

- Like charges increase energy (>0)
- opposite charges decrease energy (<0)

Must, in principle, must add up all amplitudes that give indistinguishable results:



amplitude $\propto (-\sqrt{\alpha})^4 = \underbrace{\alpha^2 \ll \alpha}$ (e+m)

justification for ignoring

others $O(\alpha^2)$?  etc.

Here, we'll focus on lowest order, but even then, sometimes there will be multiple diagrams to add up.

Probabilities $\propto |\text{Amplitude}|^2$

$\uparrow$ must sum up all amplitudes that are indistinguishable prior to squaring.

Rates for processes, including decay rates, and rates of scattering (cross sections) are $\propto |A|^2$

Antimatter

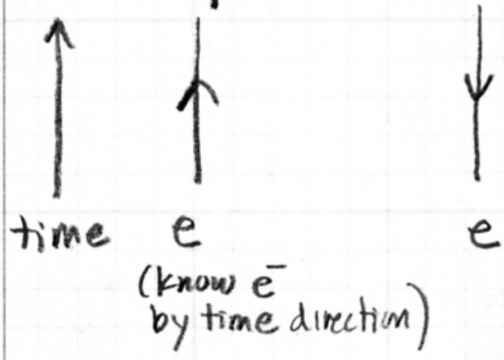
$$E = \pm\sqrt{(mc^2)^2 + (c|\vec{p}|)^2} \quad \text{as earlier noted}$$

But all matter would fall into $E<0$ states if they really existed. Quantum mechanics, however needs these states to provide a <u>complete</u> set of states

Interpretation: − sign transferred from E to t

QM: wave function $\propto e^{\frac{-i(-E)t}{\hbar}} = e^{\frac{-iE(-t)}{\hbar}}$

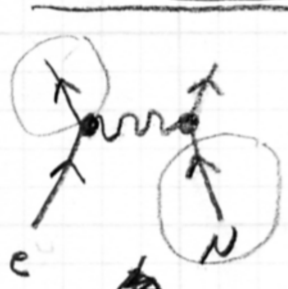
So the mathematics of $E<0$ can be satisfied if there are states that "move backward in time". These states are the antiparticles



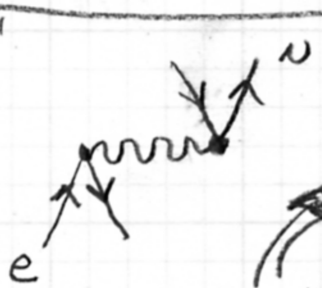
- diagram of a positron; an electron moving backward in time
- additive quantum numbers reverse sign when antimatter
  - $e^- \to e^+$ $\quad q=-1 \to q=+1$ (electric charge)
  - Lepton Number: $e^- \to +1$, $e^+ \to -1$
  - Quark # (aka, Baryon # $\frac{1}{3}$, $-\frac{1}{3}$): $q \to +1$, $\bar{q} \to -1$
  - Color: $q(R) \to \bar{q}(\bar{R})$

- γ, Z^0 have no quantum #'s that reverse; they are their own antiparticles
- $W^+ \leftrightarrow W^-$
- $g(B\bar{R}) \leftrightarrow g(R\bar{B})$

Use <u>Crossing</u> to get New Processes:



"Fold" these over



or

$e^+e^- \to \nu^+\nu^-$

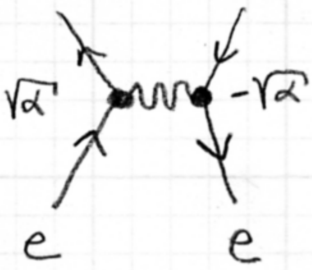
Photons "simultaneous" in time (spacelike) described similarly with "timelike" (Feynman)

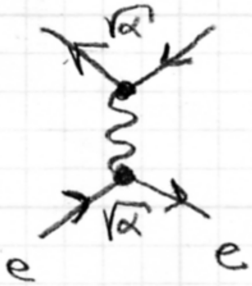
T-channel scattering
$\Rightarrow$ favors glancing blows; Rutherford Scattering $\propto 1/\sin^4\theta/2$

"S-channel"
system forgets its initial direction

$e^+e^- \to e^+e^-$ "Bhabha Scattering"

Two indistinguishable routes





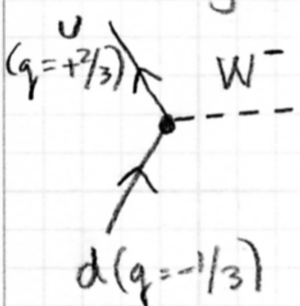
$A_t \propto O(\alpha)$
t-channel

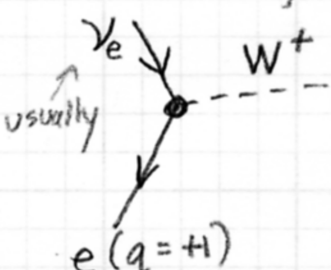
$A_s \propto O(\alpha)$
s-channel

rate $\propto |A_s - A_t|^2$

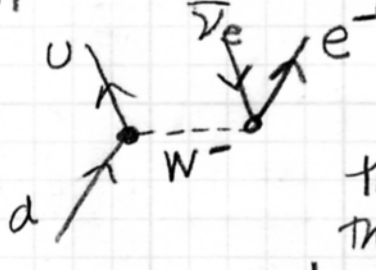
Weak Interactions

"Charged Vertex" drives β decay.



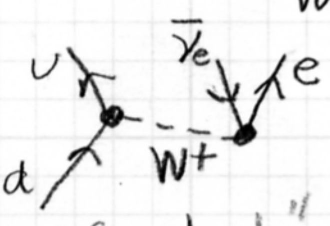


combine:



$d \to u e^- \bar{\nu}_e$

the "time ordering" of the $d \to u$ + the $\bar{\nu}_e e^-$ is not unique.. the charge of the W is not unique

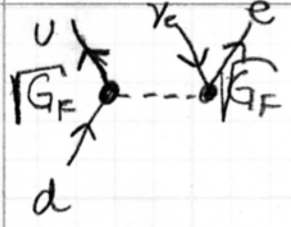


"Coupling constant" depends not just on the "fundamental coupling"... for weak interaction this is $\sqrt{\alpha}$ too! .. the "unification" of electromagnetism + weak interaction

the W mass matters too, though:

$$\text{coupling constant} \propto \frac{\sqrt{\alpha}}{M_W} \simeq \sqrt{G_F} \leftarrow \text{"Fermi Coupling"}$$

qualitative



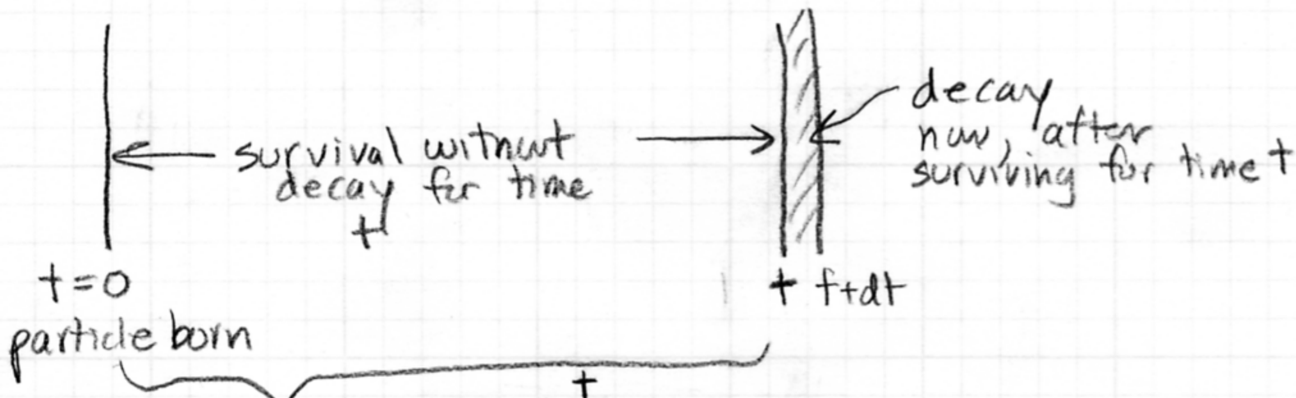
$A \propto G_F$, rate $\propto |A|^2 \propto G_F^2$

→ the decay rate is proportional to G_F^2.

An aside on decay of unstable particles

the probability of decaying in time interval dt is Γdt, where Γ = rate of decay ... really "total" rate of decay.

Decays are destructive; the interesting situation is:



$$\text{Probability} = \underbrace{(1-\Gamma dt)^{\frac{t}{dt}}}_{\text{in limit } e^{-\Gamma t}} \times \Gamma dt$$

in limit $\boxed{e^{-\Gamma t} \times \Gamma dt}$ note $\int_0^\infty \Gamma e^{-\Gamma t} dt = \underline{1}$

another way of deriving exponential decay law:

P = probability of survival

$$dP = \underbrace{(-\Gamma dt)}_{\text{loss of probability}} \underbrace{P}_{\text{existing prob}}$$

$$\int \frac{dP}{P} = -\Gamma t + C$$

$$\ln P = -\Gamma t + C$$

$$P = P_0 e^{-\Gamma t}, \quad t=0, P=1$$

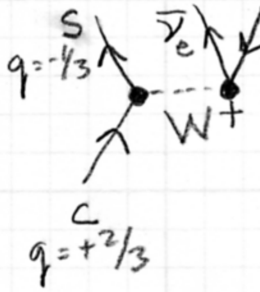
survival prob → $\boxed{P = e^{-\Gamma t}}$

if only one Feynman diagram dominates the decay, then $\propto |A|^2$

note the mean life, $\tau \equiv \int_0^\infty t e^{-\Gamma t} dt) = \frac{1}{\Gamma}$

Branching Ratio

c quark has mass sufficiently high to allow both:



$A_e \propto G_F$

$A_\mu \propto G_F$

These are distinguishable final states...

total decay rate $= \Gamma \propto |A_e|^2 + |A_\mu|^2 = \Gamma_e + \Gamma_\mu$

$\Rightarrow$ $\cong$ twice as big as it would be if <u>only</u> e or μ present

---

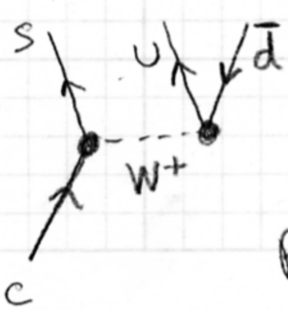
Branching Ratio is the fraction of decays that go to a specific final state.

Example above:
$$B_e = \frac{|A_e|^2}{|A_e|^2+|A_\mu|^2} = \frac{\Gamma_e}{\Gamma_e+\Gamma_\mu} = 50\%$$
$$B_\mu = \frac{|A_\mu|^2}{|A_e|^2+|A_\mu|^2} = \frac{\Gamma_\mu}{\Gamma_e+\Gamma_\mu} = 50\%$$

Simplest particles with c-quark in them are the $c\bar{u}$ (D^0) and $c\bar{d}$ (D^+). Branching ratios to $e + \mu$ have been measured...

$$\left.\begin{array}{lll} D^+ \to & e^+ \text{ anything} & 17.2 \pm 1.9\% \\ & \mu^+ \text{ anything} & 24.2 \pm 2.8\% \end{array}\right\} \text{"consistent!"}$$
$$\begin{array}{lll} D^0 \to & e^+ \quad '' & 6.75 \pm 0.29\% \\ & \mu^+ \quad '' & 6.6 \pm 0.8\% \end{array}$$

these are not 50% each! Why? There is another weak process

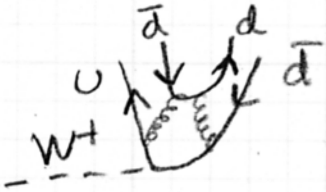


$\leftarrow B\bar{B} + R\bar{R} + G\bar{G}$

$A_{u\bar{d}} \propto G_F$ too $\times$ 3

Predict: $$B_e = \frac{\Gamma_e}{\Gamma_e + \Gamma_\mu + \underbrace{\Gamma_{u\bar{d}}}_{\simeq 3\Gamma_e}} \sim \frac{\Gamma_e}{5\Gamma_e} \sim 20\%$$

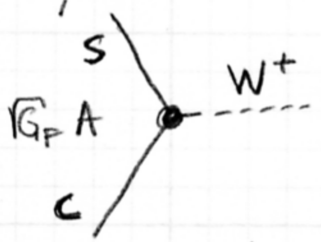
In fact, $\Gamma_{u\bar{d}}$ can be even bigger, due to the effect of gluons. Also:



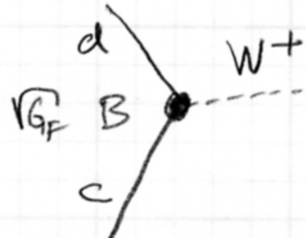
$\leftarrow$ get $\pi^+\pi^0$
not just $\pi^+ = u\bar{d}$

Cabibbo Matrix

Quark transitions via emission of W don't stay within the same "row" of the periodic table!



"favored"



"suppressed"

(two generations)

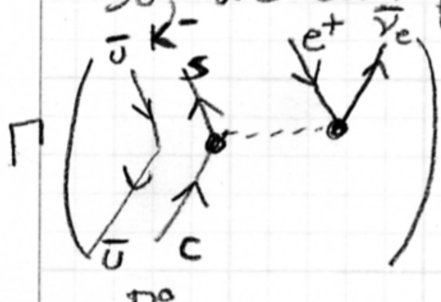
distinguishable, but $G_F[A^2+B^2] = G_F \cdot 1$

$$A^2+B^2=1$$

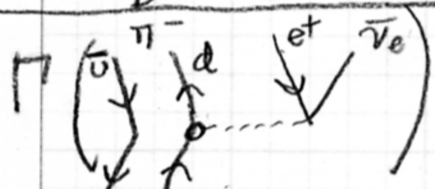
$$A = \cos\theta \qquad B = \sin\theta$$

angle called "Cabibbo" Angle. $\sin\theta \simeq 0.22$ measured

So, we can predict:



$\propto \cos^2\theta_c$



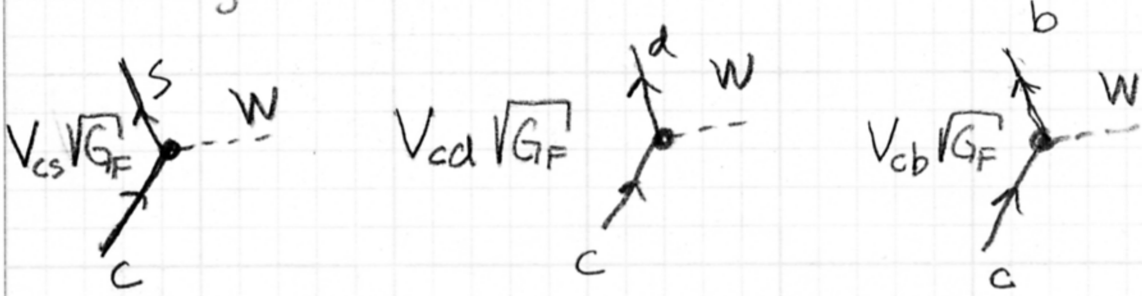
$\propto \sin^2\theta_c$

$$\simeq \frac{1-0.22^2}{0.22^2} \simeq 20$$

Data: $\dfrac{3.47\%}{.37\%} = 9.5$

difference mainly from gluon dynamics

When third quark generation discovered, idea generalized:



and $|V_{cs}|^2 + |V_{cd}|^2 + |V_{cb}|^2 = 1$

but $c \to b$ transition does not occur "spontaneously," but if the c has enough extra energy, this transition can occur.

Actually, 3x3 quarks,

$$\underset{\sim}{V} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \sim \begin{pmatrix} 0.97 & 0.22 & 0.003 \\ 0.22 & 0.97 & 0.04 \\ 0.009 & 0.04 & 0.99 \end{pmatrix} \leftarrow$$

if generations "perfect" $= \mathbb{1}$

"strength" preservation $\underbrace{\underset{\sim}{V}\,\underset{\sim}{V}^{+} = \underline{\mathbb{1}}}$

means: 3 real #s (rotation angles)
1 complex valued phase

4 parameters

3 "Cabibbo" angles
1 "Kobayashi - Maskawa" Phase

The CKM matrix