Weak Interactions

Too weak to hold together a bound state. Mainly influence:

i) "Transmutations," the change of one type of fermion into others. The most famous example is nuclear β-decay, which is driven by the process:

$$d \rightarrow u\, e^-\, \bar{\nu}_e$$

In all transmutations, it is a good idea to check whether "additive" quantum #'s are "conserved". The obvious one here is electric charge:

$$\begin{array}{ccccc} d & \rightarrow & u & e^- & \bar{\nu}_e \\ \uparrow & & \uparrow & \uparrow & \uparrow \\ Q=-\frac{1}{3} & & \underbrace{+\frac{2}{3} & -1 & 0} \end{array}$$

$-\frac{1}{3}$ net, same as started with.

Others? i) # of quarks, with quarks counting 1/3 each (not 1! why? nucleons have $\underline{3}$ quarks) and antiquarks counting −1/3, but leptons counting 0

$$\begin{array}{ccccc} d & \rightarrow & u & e^- & \bar{\nu}_e \\ \uparrow & & \uparrow & \uparrow & \uparrow \\ B = +1/3 & \rightarrow & +1/3 & 0 & 0 \end{array}$$

↑ for baryon #

ii) # of leptons; e^-, μ^-, τ^- count +1, ν_e, ν_μ, ν_τ count +1; $e^+ \mu^+ \tau^+$ count −1, $\bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau$ count −1

iii) differently: electron #, μ-#, τ-#

$e^-, \nu_e = e\# +1$, $e^+, \bar{\nu}_e = e\# -1$; $\mu, \tau = 0$

$\mu^-, \nu_\mu = \mu\# + 1$; $\mu^+\ \bar{\nu}_\mu = \mu\# - 1$; $e, \tau = 0$

$\tau^-, \nu_\tau = \tau\# + 1$; $\tau^+\ \bar{\nu}_\tau = \tau\# - 1$; $e\, \mu = 0$

"Weak" Transmutations are responsible for the instability of all fundamental fermions except $\nu_e, \nu_\mu, \nu_\tau, e^-, u, + d$, which are stable

1) "Transmutations" continued

The process $d \to u e^- \bar{\nu}_e$ is by itself not observable, because quarks in normal matter are bound into nucleons. What is seen is:

$$\left.\begin{array}{l} n \to p e^- \bar{\nu}_e \\ \uparrow \\ (udd) \to (uud)\, e^- \bar{\nu}_e \end{array}\right\}$$

mean life $\tau = 886.7 \pm 1.9\,s$

$c\tau = 2.7 \cdot 10^8\,km$

$\ggg 10^{-15}\,m$ nucleon size

"long" life, <u>weak</u> interaction

Even neutrons are relatively unstable; a more "practical" example is:

$$^{40}K \longrightarrow {}^{40}C\, e^- \bar{\nu}_e \qquad \tau = 1.28 \cdot 10^9 \text{ years}$$

^{40}K: $Z = 19$, $N = 21$

^{40}C: $Z = 20$, $N = 20$

However 10% of the time,

$$^{40}K + e^- \to {}^{40}A\, \nu_e \qquad \text{"electron capture"}$$

e^-: atomic s-wave

^{40}A: $Z = 18$, $N = 22$

2) "Scattering" sort of like electron capture, only the analog of the electron now has "extra" energy; to keep focus on the <u>weak</u> interaction, consider

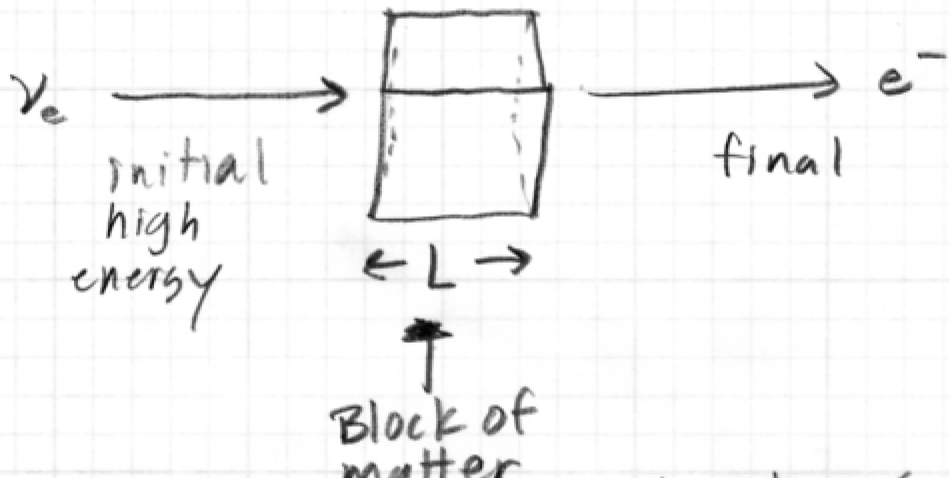
$$\nu_e n \to p e^- \qquad (\text{"cross" the } \bar{\nu}_e \text{ from } n \to p e^- \bar{\nu}_e)$$

For $E_{\nu_e} \gg |m_p + m_{e^-} - m_n|$

the cross section for this scattering process is $\simeq \underline{3 \cdot 10^{-43}\,m^2}$

that is the key # that quantifies the weak interaction.

What is a cross section?



say, neutrons, N neutrons$/m^3$

The "process" is $\nu_e n \to p e^-$ (really, $\nu_e X \to Y e^-$)

"Probability" $P \propto L$

$\propto N$

$\therefore P \propto NL$ units? $\frac{\#}{m^3} \cdot m = \frac{\#}{m^2}$

$P = \underset{m^2}{\sigma} \cdot \underset{\#/m^3}{N} \underset{m}{L} \quad \propto \#$

makes sense as long as $P \ll 1$

Consider: $\boxed{\nu_e \text{ passing through the earth.}}$

$L \to \underset{R_E}{\underset{\parallel}{6.38 \cdot 10^6}}\ m$ (at most)

$N \to \sim \frac{1}{2}$ of the nucleons are neutrons (others protons)

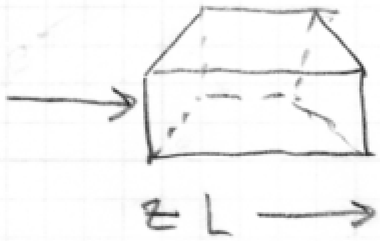
$= \frac{1}{2} \cdot \frac{3.57 \cdot 10^{51}}{\frac{4\pi}{3} \cdot R_E^3}$ ← last lecture

$N = 1.64 \cdot 10^{30}\ 1/m^3$ $\left(\simeq 1/(10^{-10} m)^3\right)$

$P = 3 \cdot 10^{-43} \cdot 1.64 \cdot 10^{30} \cdot 6.38 \cdot 10^6 = \underline{\underline{3 \cdot 10^{-6}}}$

Cross Sections $\longleftrightarrow$ Feynman Diagrams

what if,



$$P = \sigma \cdot N \cdot L > 1 \; ?$$

σ = cross section
$N = \#/\text{Volume}$

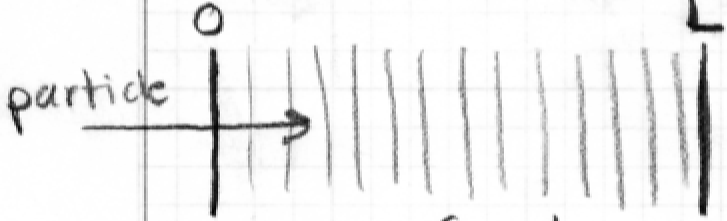
is "event" destructive or non-destructive? $\nu_e n \to p e^-$ is destructive: ν_e is destroyed.
But if the event is non-destructive, then multiple events could happen.

Example: $e^- + \text{Atom} \to \underbrace{\text{Atom}^*}_{\text{excited}} + e^-$

If the event is non-destructive, then we re-interpret P as the mean # of events that occur in the material of length L.

$$\cancel{P} \to \mu = \sigma N L \qquad \mu = \text{mean } \#$$

Justification? Let's first compute the probability of having 0 events, by splitting the slab into infinitesimal layers



$\to \; \leftarrow \; \delta x = \frac{L}{M} \leftarrow$ a big integer, = # of layers

in one layer, the probability of an event is:

$$\rho = \sigma \cdot N \delta x = \frac{\sigma N L}{M} = \frac{\mu}{M}$$ (can be made arbitrarily small, $M \to \infty$)

so the probability of no event is:

$$1 - \rho = 1 - \frac{\mu}{M}$$

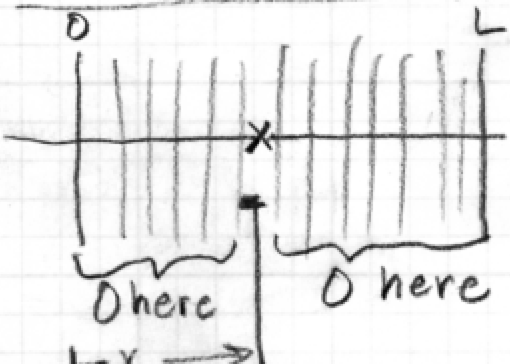
Again, that is in <u>one</u> layer. The probability of no events in <u>any</u> of the layers will be:

$$\mathcal{L}(0) = (1-p)^M = \left(1-\frac{\nu}{M}\right)^M \quad \text{take } M\to\infty$$

$$\boxed{\mathcal{L}(0) = e^{-\nu}} \qquad \nu = \sigma N L$$

Even if $\nu > 1$, this makes sense ... whereas for the probability of 0 is $1-\nu$ by itself is <u>non</u>-sense when $\nu > 1$.

<u>What about the probability of exactly 1 event?</u>



$\Leftarrow$ but 0 everywhere else!

$\downarrow$

probability is

$$e^{-\sigma N(L-dx_1)}$$

$$= e^{-\nu\left(1-\frac{dx_1}{L}\right)}$$

1 there, $p = \frac{\nu}{L}\,dx_1$

$= \sigma N\, dx_1$

is probability

So, the probability of 1 event at x_1 will be the product:

$$d\mathcal{L}(1) = \left(\frac{\nu}{L}\,dx_1\right)\left(e^{-\nu\left(1-\frac{dx_1}{L}\right)}\right)$$

as $dx_1 \to 0$, contribution vanishes.

And, to get the probability of exactly 1 event anywhere in the slab will be:

$$\boxed{\mathcal{L}(1) = \int_0^L d\mathcal{L}(1) = \left(\frac{\nu}{L}e^{-\nu}\right)\int_0^L dx_1 = \nu e^{-\nu}}$$

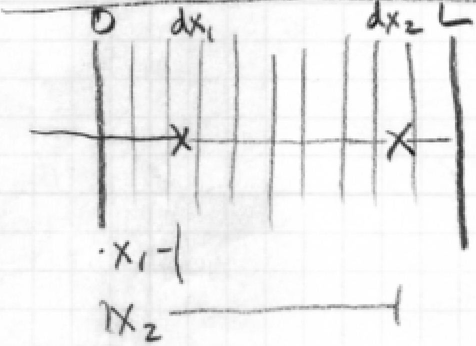
note, this is OK if $\nu > 1$ too! That is, I assert $\mathcal{L}(1) < 1$ always.

If the event is destructive, the $\mathcal{L}(2)$ is (physically) forbidden, and one must renormalize:

$$\mathcal{L}_D(0) = \frac{\mathcal{L}(0)}{\mathcal{L}(0)+\mathcal{L}(1)} = \frac{1}{1+\nu} < 1$$

$$\mathcal{L}_D(1) = \frac{\mathcal{L}(1)}{\mathcal{L}(0)+\mathcal{L}(1)} = \frac{\nu}{1+\nu} < 1$$

How about the probability of exactly 2 events (non-destructive)?



$$d^2\mathcal{L}(2) = \left[\frac{\nu}{L}dx_1\right]\left[\frac{\nu}{L}dx_2\right] e^{-\nu} \times \frac{1}{2!}$$

first interaction in dx_1 at x_1

second at x_2 in dx_2

$\simeq \mathcal{L}(0)$ everywhere else

assuming the two are indistinguishable

$$\mathcal{L}(2) = \int_0^L\!\!\int_0^L d^2\mathcal{L}(2)\,dx_1\,dx_2$$

$$\boxed{\mathcal{L}(2) = \frac{1}{2!}\nu^2 e^{-\nu}}$$

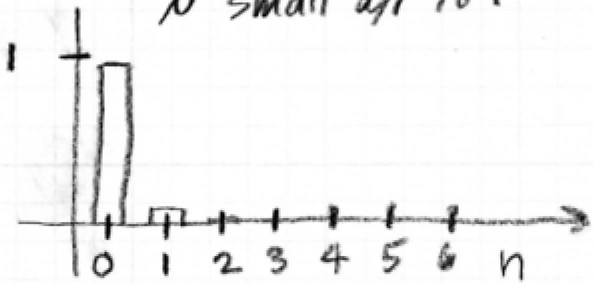
finally,

$$\boxed{\mathcal{L}(n) = \frac{1}{n!}\nu^n e^{-\nu}}$$

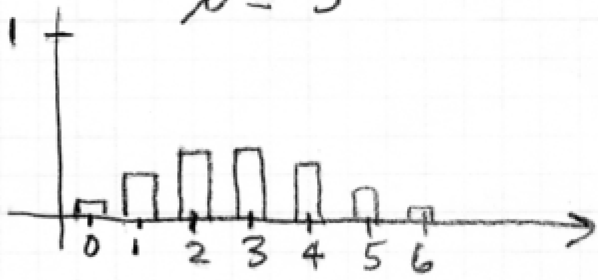
← "Poisson" distribution

- parameter ν (continous)
- function of n (discrete)

ν small w/r to 1



$\nu = 3$



mean #:

$$\langle n \rangle = \frac{\sum_{n=0}^{\infty} n \mathcal{L}(n)}{\sum_{n=0}^{\infty} \mathcal{L}(n)}$$

<u>Denominator</u>: $\sum_{n=0}^{\infty} \mathcal{L}(n) = e^{-\nu} \underbrace{\sum_{n=0}^{\infty} \frac{\nu^n}{n!}}_{e^{+\nu}} = e^{-\nu} \cdot e^{+\nu} = 1$

<u>Numerator</u>: $\sum_{n=0}^{\infty} n \mathcal{L}(n)$

$$= e^{-\nu} \sum_{n=0}^{\infty} \frac{n \cdot \nu^n}{n!} = \nu e^{-\nu} \sum_{n=1}^{\infty} \frac{\nu^{n-1}}{(n-1)!}$$

$$= \nu e^{-\nu} \cdot e^{+\nu} = \nu$$

so $\boxed{\langle n \rangle = \nu}$

The bottom line: when ν is small ($\ll 1$), only $1-\nu$ = prob 0, ν = prob 1 matter. <u>Higher order</u> terms matter as $\nu \sim 1$.

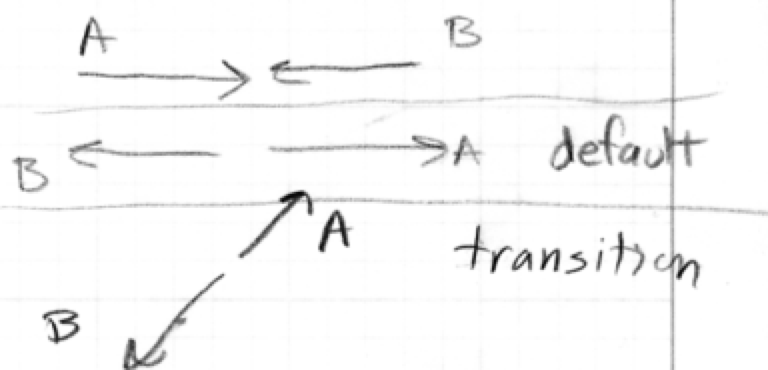
The Concept of Feynman Diagrams

① "Expansion" in a small # like the ν in the poisson distribution
Get the "lowest order" physics.
Related to QM perturbation theory

② Draw pictures: • fermions always "solid" lines – net # conserved
• bosons usually wiggly, dotted, dashed, etc – net # not conserved.

③ Usually represent transitions from the "default":

ⓐ Scattering



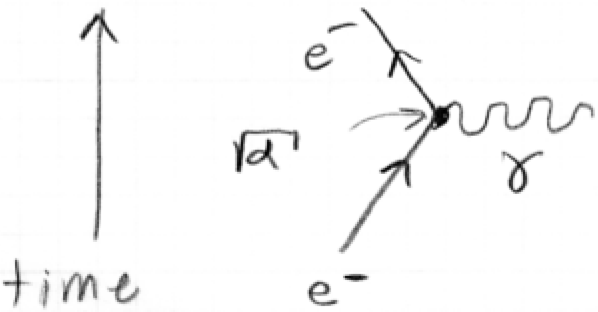
ⓑ Decay $\overset{d}{\bullet}$ → stays a d (default)
→ $u e^- \bar{\nu}_e$ (transition)

ⓒ Perturbation

④ Build up complex situations out of simple "vertices.

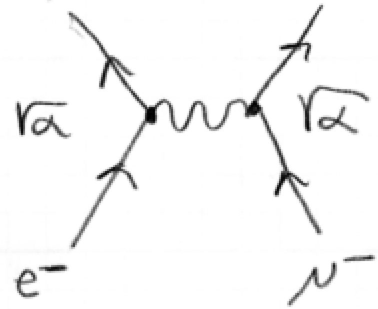
⑤ "Rules" at each vertex, which depend on the type of interaction.

Electromagnetic Vertex (everything with charge!)

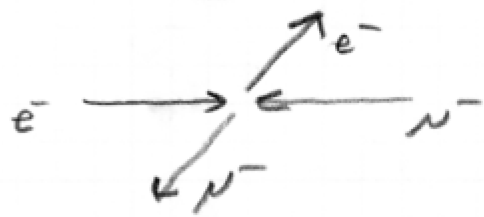


does ~~not~~ describe
$e^- \to e^- \gamma$
won't conserve energy and momentum.

but:

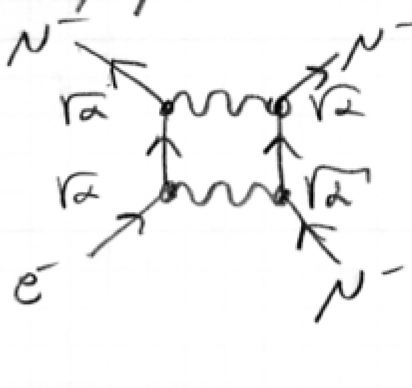


does describe



Q.M. amplitude $\propto$ perturbing Hamiltonian is $\propto (\sqrt{\alpha})^2 \propto \alpha \sim 1/137$

The smallness of α, (like the smallness of ν) means more complex processes, like:



perturbation is $\propto (\sqrt{\alpha})^4$ or $1/137$ times smaller than the previous.

Now: if $\alpha \gg 1$ (like the strong interaction) you can see how "higher" orders could be important, similar to Poisson having $\nu \gg 1$.