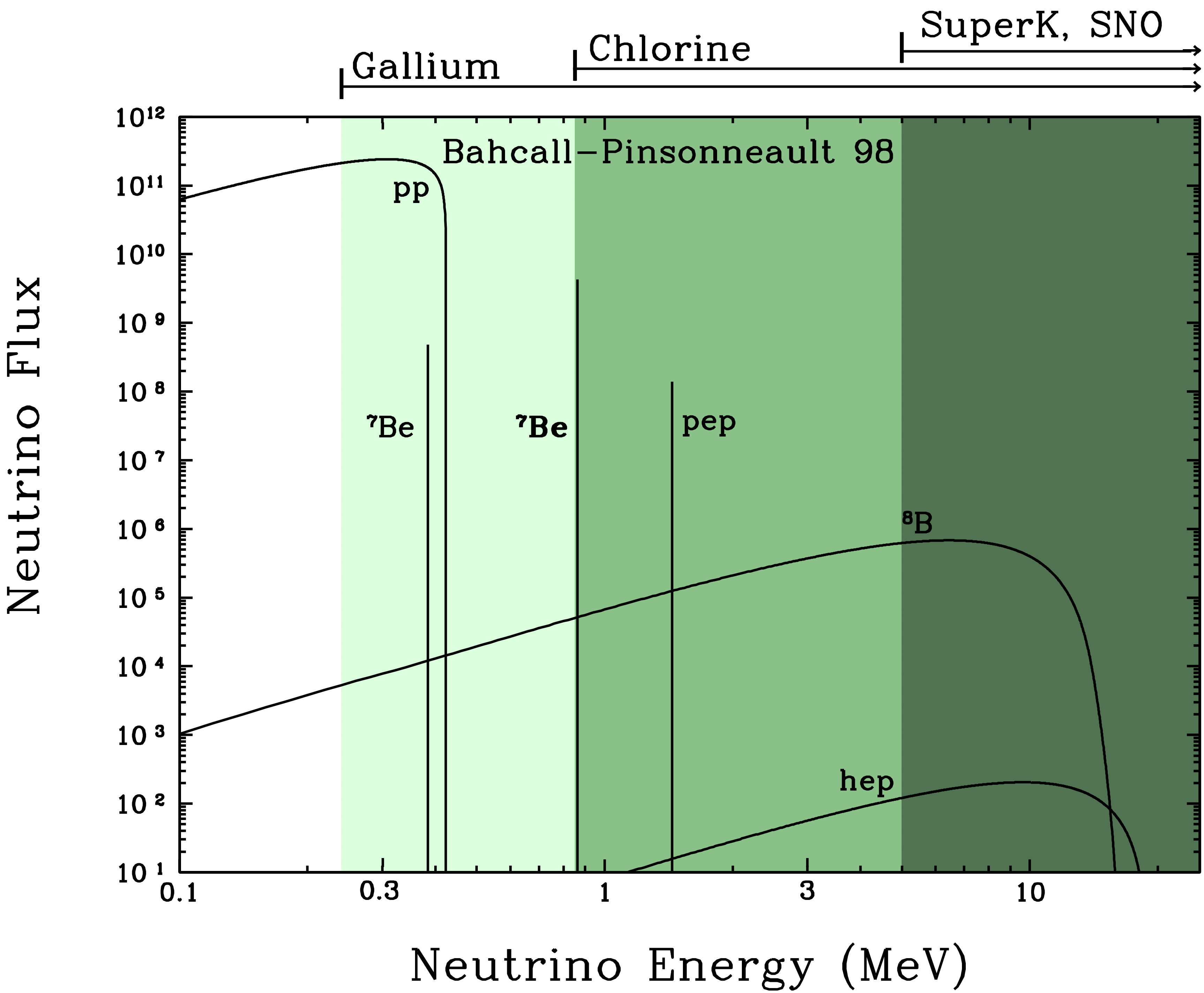
REACTION	TERM (%)	ν ENERGY (MeV)
$p + p \rightarrow {}^2H + e^+ + \nu_e$	(99.96)	≤ 0.423
or		
$p + e^- + p \rightarrow {}^2H + \nu_e$	(0.44)	1.445
${}^2H + p \rightarrow {}^3He + \gamma$	(100)	
${}^3He + {}^3He \rightarrow \alpha + 2p$	(85)	
or		
${}^3He + {}^4He \rightarrow {}^7Be + \gamma$	(15)	
${}^7Be + e^- \rightarrow {}^7Li + \nu_e$	(15)	$\begin{cases} 0.863 & 90\% \\ 0.385 & 10\% \end{cases}$
${}^7Li + p \rightarrow 2\alpha$		
or		
${}^7Be + p \rightarrow {}^8B + \gamma$	(0.02)	
${}^8B \rightarrow {}^8Be^* + e^+ + \nu_e$		< 15
${}^8Be^* \rightarrow 2\alpha$		
or		
${}^3He + p \rightarrow {}^4He + e^+ + \nu_e$	(0.00003)	< 18.8

Neutrino terminations from BP2000 solar model. Neutrino energies include solar corrections: J. Bahcall, Phys. Rev. C, 56, 3391 (1997).


Gallium
Chlorine
SuperK, SNO
Bahcall–Pinsonneault 98
pp
^{7}Be
^{7}Be
pep
^{8}B
hep
Neutrino Flux
Neutrino Energy (MeV)
0.1
0.3
1
3
10
10^{12}
10^{11}
10^{10}
10^{9}
10^{8}
10^{7}
10^{6}
10^{5}
10^{4}
10^{3}
10^{2}
10^{1}

Neutrino Mixing

- Schrodinger Equation in the $m \sim 0$ limit:

$$H = E = c\sqrt{(mc)^2 + \vec{p}^2} \simeq c|\vec{p}| + \frac{m^2c^3}{2|\vec{p}|}$$

→ eigenstates of momentum will again be eigenstates of energy

$$\psi(\vec{x},t) \propto \psi(t)\, e^{i\vec{p}\cdot\vec{x}/\hbar}$$

then $i\hbar \frac{\partial}{\partial t}\psi(t) = \left(c|\vec{p}| + \frac{m^2c^3}{2|\vec{p}|}\right)\psi(t)$

$$\psi(x,t) = \underbrace{e^{\left(-\frac{c|\vec{p}|t}{\hbar} + \frac{\vec{p}\cdot\vec{x}}{\hbar}\right)i}}_{\text{line of constant phase (in direction } \vec{p})} \; e^{\frac{-im^2c^3}{2|\vec{p}|\hbar}t}$$

$+\frac{c|\vec{p}|t}{\hbar} = \frac{|\vec{p}|x}{\hbar}$

or $x/t = c$ (movement at speed of light)

→ "phase lag" due to non-zero mass

- When a neutrino of one flavor (say ν_e) evolves in time, it appears that eventually a neutrino of one other flavor appears (say ν_μ) (Technically, all 3 neutrinos could get in the act, but so far, it appears that considering pairs of ν's is sufficient).

The vacuum appears to cause the transmutation. The situation resembles that of birefringent crystal, where, for example, circular polarizations might be the eigenstates (not linear polarizations), and left-handed and right-handed polarizations might propagate with different speeds.

- incorporate the evolution via a "mass-squared" matrix:

one component (no mixing) $i\hbar \frac{\partial}{\partial t}\psi(t) = \left(c|\vec{p}| + \frac{m^2c^3}{2|\vec{p}|}\right)\psi(t)$

neutrino masses (not charged lepton)

two component: $i\hbar \frac{\partial}{\partial t}\begin{pmatrix}\psi_e \\ \psi_\mu\end{pmatrix} = \left[c|\vec{p}|\,\underset{\sim}{\mathbb{1}} + \frac{c^3}{2|\vec{p}|}\underbrace{\begin{pmatrix} m_e^2 & m_{e\mu}^2 \\ m_{e\mu}^2 & m_\mu^2\end{pmatrix}}\right]\begin{pmatrix}\psi_e \\ \psi_\mu\end{pmatrix}$.

we know how to diagonalize!

$$\begin{pmatrix} m_e^2 & m_{e\mu}^2 \\ m_{e\mu}^2 & m_\mu^2\end{pmatrix} = \frac{1}{2}(m_e^2+m_\mu^2)\underset{\sim}{\mathbb{1}} + \frac{1}{2}(m_e^2-m_\mu^2)\,\underset{\sim}{\tau}_z + m_{e\mu}^2\,\underset{\sim}{\tau}_x$$

$$\underset{\text{convention}}{\tan 2\theta} = \frac{m_{e\mu}^2}{\frac{1}{2}(m_e^2-m_\mu^2)}$$

- 2θ small: $m_{e\mu}^2 \ll \left|\frac{1}{2}(m_e^2-m_\mu^2)\right|$
- $2\theta \sim \frac{\pi}{2}$, $\theta \sim \pi/4$ $m_{e\mu}^2 \gg \left|\frac{1}{2}(m_e^2-m_\mu^2)\right|$

eigenvalues:

$$m_1^2 = \frac{1}{2}(m_e^2+m_\mu^2) - \sqrt{\frac{1}{4}(m_e^2-m_\mu^2)^2 + m_{e\mu}^4} \Rightarrow |\nu_1\rangle = \cos\theta|\nu_e\rangle + \sin\theta|\nu_\mu\rangle$$

$$m_2^2 \; \frac{1}{2}(m_e^2+m_\mu^2) + \sqrt{\frac{1}{4}(m_e^2-m_\mu^2) + m_{e\mu}} \Rightarrow |\nu_2\rangle = -\sin\theta|\nu_e\rangle + \cos\theta|\nu_\mu\rangle$$

- At $t=0$, imagine a pure ν_e produced, like in the sun. Need to expand the pure ν_e in a superposition of eigenstates:

$$\begin{matrix}\nu_e \\ \nu_\mu\end{matrix} \rightarrow \begin{pmatrix}1 \\ 0\end{pmatrix} = \alpha\begin{pmatrix}\cos\theta \\ \sin\theta\end{pmatrix} + \beta\begin{pmatrix}-\sin\theta \\ \cos\theta\end{pmatrix}$$

so, $(\cos\theta \;\; \sin\theta)\begin{pmatrix}1 \\ 0\end{pmatrix} = \cos\theta = \alpha\times(\cos^2\theta + \sin^2\theta) + \beta\times(0)$

$$\alpha = \cos\theta$$

similarly, $\beta = -\sin\theta$

then $\begin{pmatrix}\psi_e(t) \\ \psi_\mu(t)\end{pmatrix} = \left[\cos\theta\begin{pmatrix}\cos\theta \\ \sin\theta\end{pmatrix} e^{\frac{-im_1^2c^3}{2|\vec{p}|\hbar}t} - \sin\theta\begin{pmatrix}-\sin\theta \\ \cos\theta\end{pmatrix} e^{\frac{-im_2^2c^3}{2|\vec{p}|\hbar}t}\right] e^{\left(\frac{-c|\vec{p}|t}{\hbar} + \frac{\vec{p}\cdot\vec{x}}{\hbar}\right)}$, To lowest order, it is OK to replace:

$$c|\vec{p}| = E \qquad t = L/c = |\vec{x}|/c$$

Then

$$|\psi_\mu(L)|^2 = \left|\sin\theta\cos\theta\left[e^{\frac{-i m_1^2 c^3 L}{2E\hbar}} - e^{\frac{-i m_2^2 c^3 L}{2E\hbar}}\right]\right|^2$$

$$\left|e^{-iAL} - e^{-iBL}\right|^2 = 2 - 2\,\mathrm{Re}\left(e^{-iAL}e^{+iBL}\right)$$

$$= 2 - 2\cos((A-B)L) = 4\sin^2\tfrac{1}{2}(A-B)L$$

$$\boxed{|\psi_\mu(L)|^2 = \underbrace{4\sin^2\theta\cos^2\theta}_{\sin^2 2\theta}\ \sin^2\left[\frac{(m_2^2-m_1^2)c^3L}{4E\hbar}\right] \quad \text{PLOT}}$$

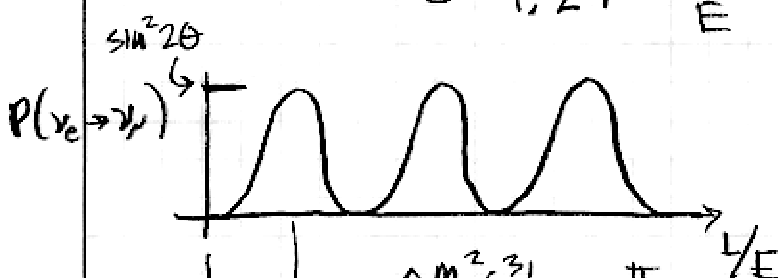
numerics: generally, put $(m_2^2-m_1^2)c^4$ in eV^2, $\equiv \Delta M^2c^4$
L/E in km/GeV or m/MeV

$$\hbar c = 197.3\ \text{MeV-fm} = 197.3\ \text{MeV}\cdot 10^{-15}\,\text{m}$$

$$\frac{\Delta M^2c^4L}{4E\hbar c} = \frac{1}{4\cdot 197.3\cdot 10^{-15}}\ \underbrace{\frac{1}{10^{12}}}_{(\text{MeV}/\text{eV})^2} \times \frac{\Delta M^2c^4\,L}{E}$$

$$= 1.27\,\frac{\Delta M^2c^4L}{E}$$

with ΔM^2c^4 in eV^2
L/E in $\frac{m}{MeV}$ $\frac{km}{GeV}$

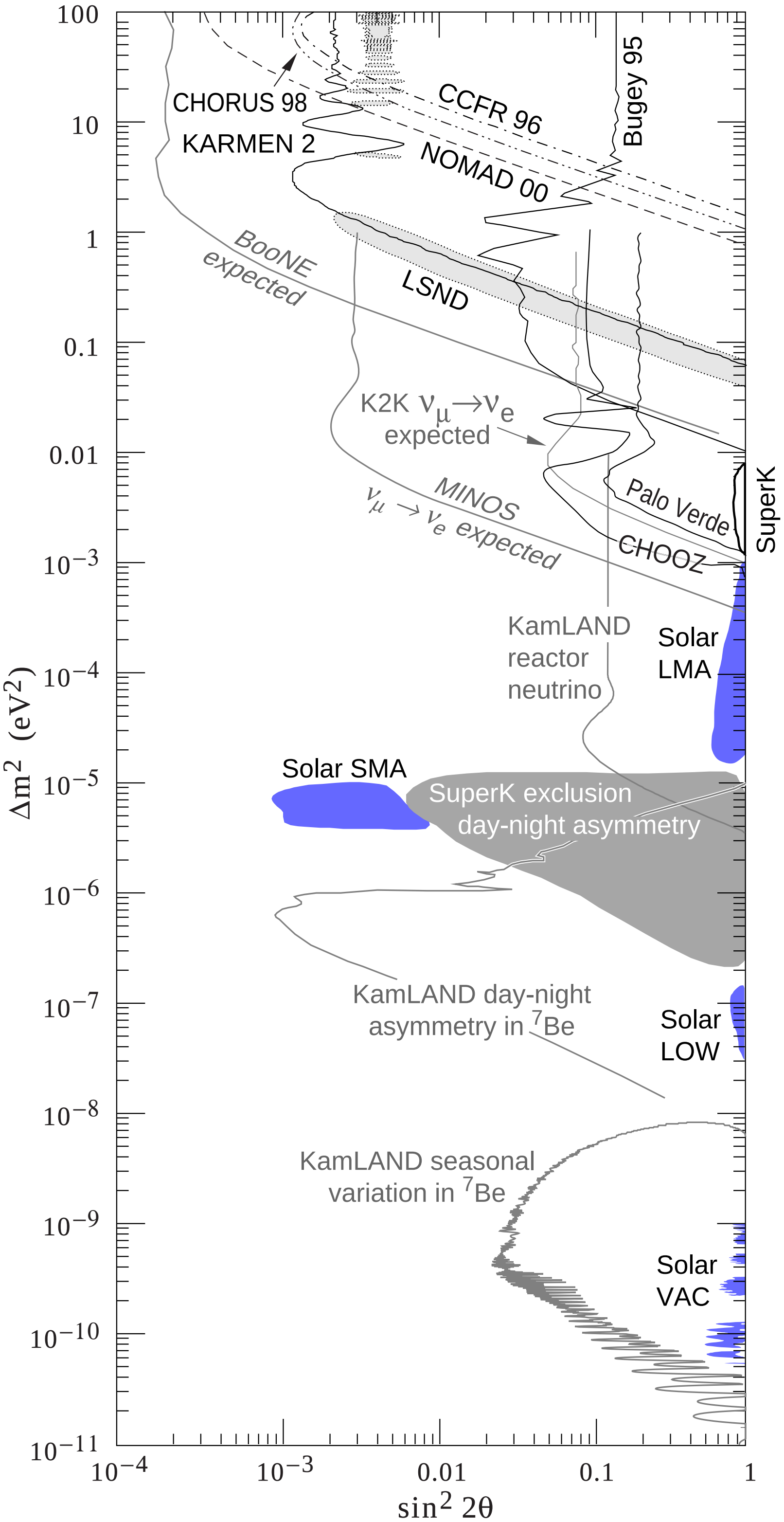


$$\frac{\Delta M^2c^3L}{4E\hbar} = \frac{\pi}{2}$$

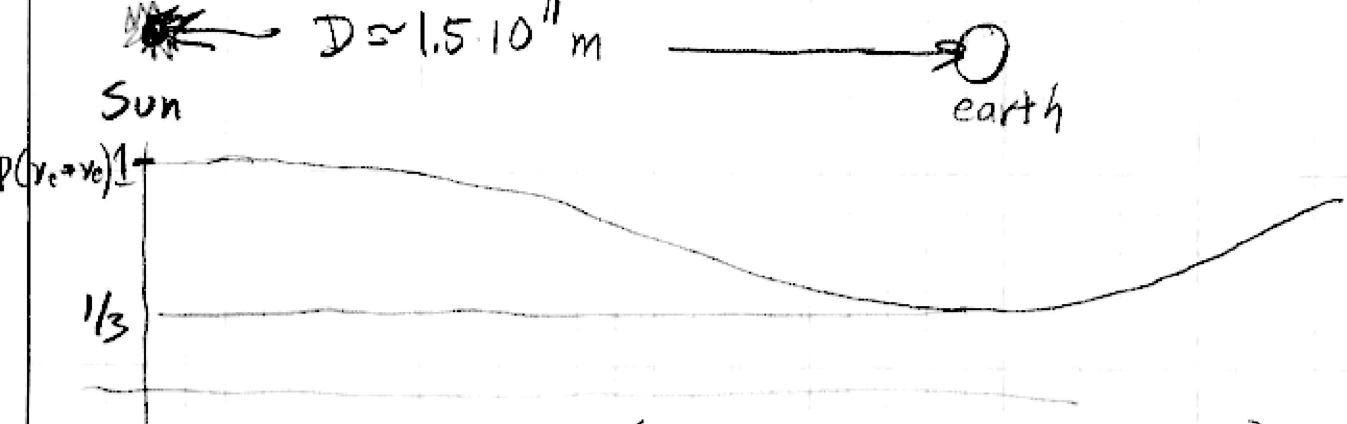
starting in $\simeq$ 1965,
an experiment based
on $\nu_e + {}^{37}Cl \to {}^{37}Ar + e^-$
sun
found $\simeq 0.33 \pm 0.06$
as many ν_e as expected

TWO-FLAVOR OSCILLATION PARAMETERS AND LIMITS

Written April 2000 by H. Murayama (LBNL).



one possibility:



$E_\nu \simeq 7$ MeV (^8B, detected by Chlorine)

$$1.27 \, \frac{\Delta m^2 c^4 \cdot 1.5 \cdot 10^{11} \text{ m}}{7 \text{ MeV}} \sim \frac{\pi}{2}$$

$$\Delta m^2 \sim 6 \cdot 10^{-11} \text{ eV}^2$$

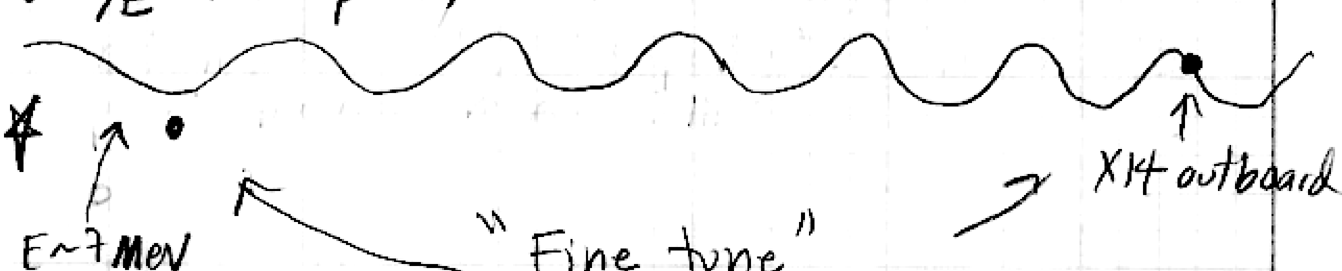
(Homework: maybe as low as $\sim 2.3 \cdot 10^{-11}$ eV^2)

$\simeq$ 1990's, experiments using $\nu_e + {}^{71}Ga \to {}^{71}Ge + e^-$ started.

They saw: $\simeq 0.56 \pm 0.06$ as many ν_e's as expected.

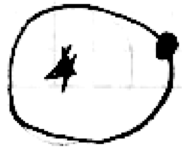
$E \sim 0.5$ MeV

- L/E now up by factor of 14



"Fine tune"

but earth sun distance has seasonal variation



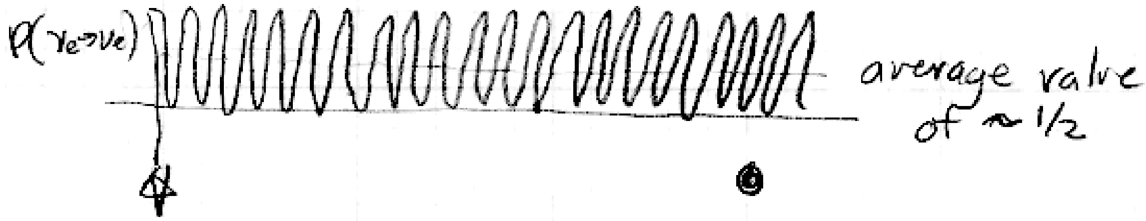
can use seasonal variation as probe (Borexino).

In $\simeq 80$'s to present, Kamiokande, Super-K, measure:

$$\# \nu_e = 0.47 \pm 0.08 \text{ expected}$$
$$E \sim 10 \text{ MeV}$$

Crudely, situation not too inconsistent with <u>much</u> larger Δm^2, and

$$\langle P(\nu_e \to \nu_e) \rangle \sim \langle P(\nu_e \to \nu_\mu) \rangle \sim 1/2$$



Know from absence of large day night asymmetry that (in S-K):

either $L = \frac{\pi}{2 \times 1.27} \frac{E}{\Delta m^2 c^4} \gg 2R_E$

$$\Delta m^2 c^4 \ll \frac{\pi}{2\,1.27} \frac{E}{2R_E} \sim 10^{-6} \text{ eV}^2$$

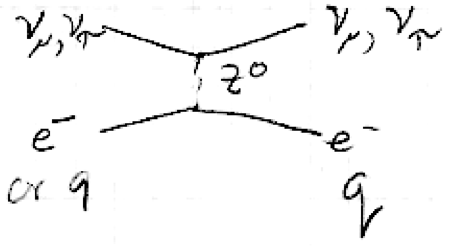
or $L \ll 2R_E$ so $\Delta m^2 c^4 \gg 10^{-6} \text{eV}^2$

(or, $\sin^2 2\theta$ very small).

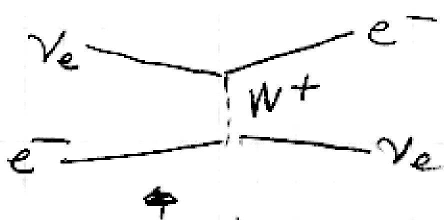
There is a favored solution these days with $\Delta m^2 c^4 \sim 10^{-4} \text{ eV}^2$

MSW effect

Only way very low energy (ν_μ, ν_τ) interact with matter is via Z^0:



ν_e has one extra diagram



ν_e gets more energy, $\sim \sqrt{2}\, G_F N_e$ ($\hbar, c = 1$)

$$\frac{G_F}{(\hbar c)^3} = 1.2\cdot 10^{-5}\,\frac{1}{\text{GeV}^2}$$

N_e = # density of electrons

$$= \frac{\rho \leftarrow \text{mass density gm/cm}^3}{\bar{A} \leftarrow \text{mean atomic weight}} \times N_A \times \bar{Z}$$

N_A ↑ Avogadro # $\bar{Z}$ ↑ mean atomic #.

$\frac{\bar{Z}}{\bar{A}} \sim \frac{1}{2}$ earth, $\frac{\bar{Z}}{\bar{A}} \sim \frac{2}{3}$ sun

$$N_e \sim 6\cdot 10^{23} \times \frac{1}{2} \times \rho\left(\frac{\text{gm}}{\text{cm}^3}\right)\left(\frac{\bar{Z}}{\bar{A}}\right) \text{ in } 1/\text{cm}^3 \qquad 10^6\,\text{cm}^3 = 1\,\text{m}^3$$

$$N_e \sim 3\cdot 10^{29}\,(1/\text{m}^3) \times (\rho \text{ in gm/cm}^3) \times \left(\frac{\bar{Z}}{\bar{A}}\right)$$

$$G_F N_e = 3\times 1.2\cdot 10^{24}\,(2\cdot 10^{-1})^3\,\frac{\text{GeV}^3\cdot\text{fm}^3}{\text{GeV}^2\cdot\text{m}^3}\,(\rho \text{ in gm/cm}^3)\left(\frac{\bar{Z}}{\bar{A}}\right)$$

$$\simeq 4\times 8\times 10^{21}\cdot 10^{-45}\times 10^9\ \text{eV} \times \rho \times \left(\frac{\bar{Z}}{\bar{A}}\right)$$

$$G_F N_e \simeq 3\cdot 10^{-14}\,\text{eV} \times \rho$$

so $\boxed{\sqrt{2}\,G_F N_e \simeq (4\cdot 10^{-14}\,\text{eV})\cdot \rho\left(\frac{\text{gm}}{\text{cm}^3}\right) \times \frac{\bar{Z}}{\bar{A}}}$

Like a δm_e^2 (not a m_ν^2) of ...

$$\frac{c^3}{2|\vec{p}|}\,\delta m_e^2 = (4\cdot 10^{-14}\,\text{eV})\cdot \rho$$

or $\boxed{\delta m_e^2 c^4 \sim 8\cdot 10^{-8}\,\text{eV}^2\cdot \rho\left(\frac{\text{gm}}{\text{cm}^3}\right) \times E_\nu(\text{MeV}) \times \frac{\bar{Z}}{\bar{A}}}$

Sun: center $\rho \sim 150$ gm/cm^3

$\bar{Z}/\bar{A} \sim \frac{2}{3}$

so $\delta m_e^2 c^2 \sim 8\cdot 10^{-6}\,\text{eV}^2 \times E_\nu(\text{MeV}) \sim 10^{-5}\,\text{eV}^2\,E_\nu(\text{MeV})$

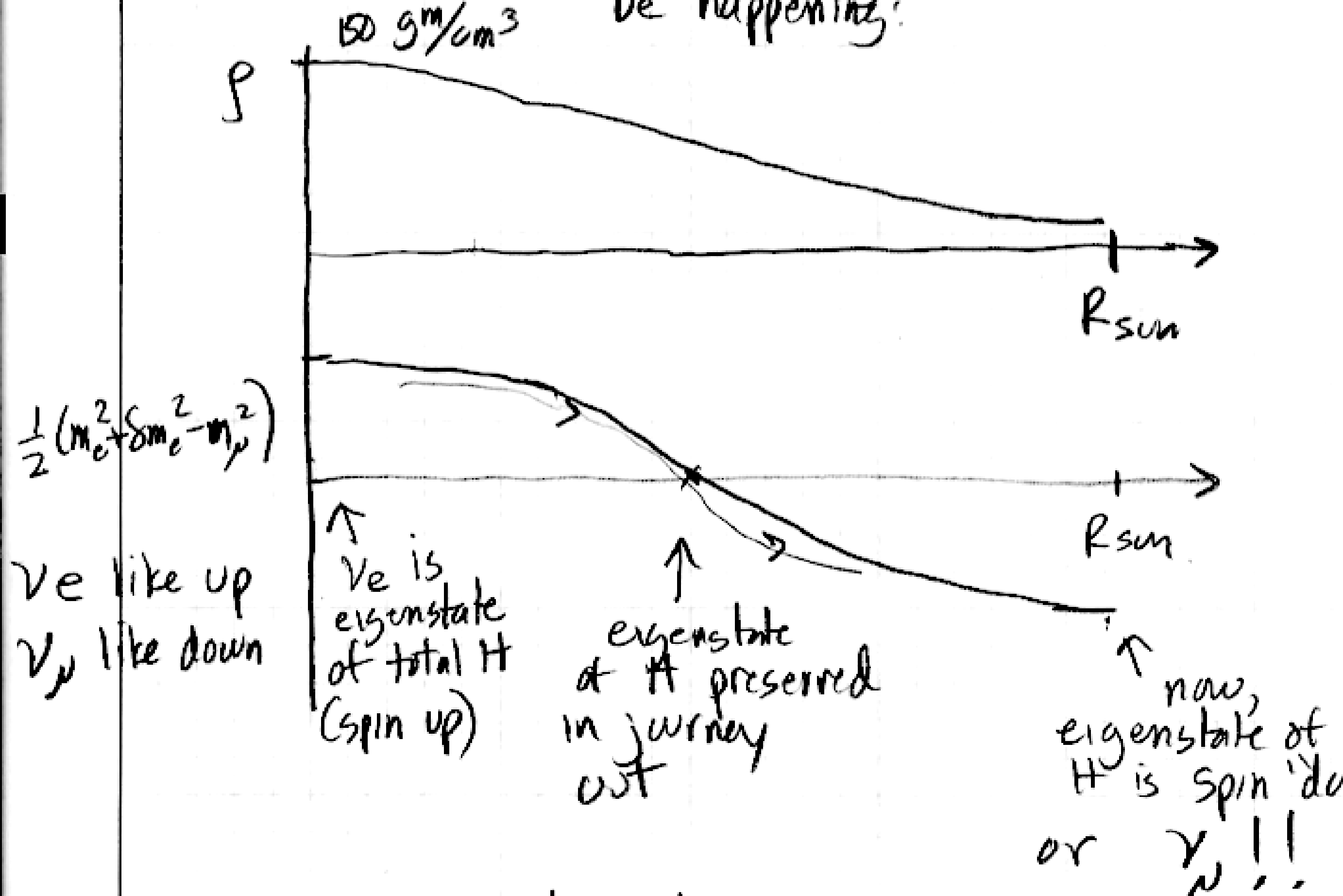
what can this do? Now Hamiltonian has an extra piece,

$$\Rightarrow \begin{pmatrix} m_e^2 + \delta m_e^2 & m_{e\mu}^2 \\ m_{e\mu}^2 & m_\mu^2 \end{pmatrix}$$

$$= \frac{1}{2}(m_e^2 + m_\mu^2 + \delta m_e^2) + \underbrace{\frac{1}{2}(m_e^2 + \delta m_e^2 - m_\mu^2)}\,\underset{\sim}{\sigma}_z + m_{e\mu}^2\,\underset{\sim}{\sigma}_x$$

In the sun, $\delta m_e^2 \neq$, $\propto \rho$

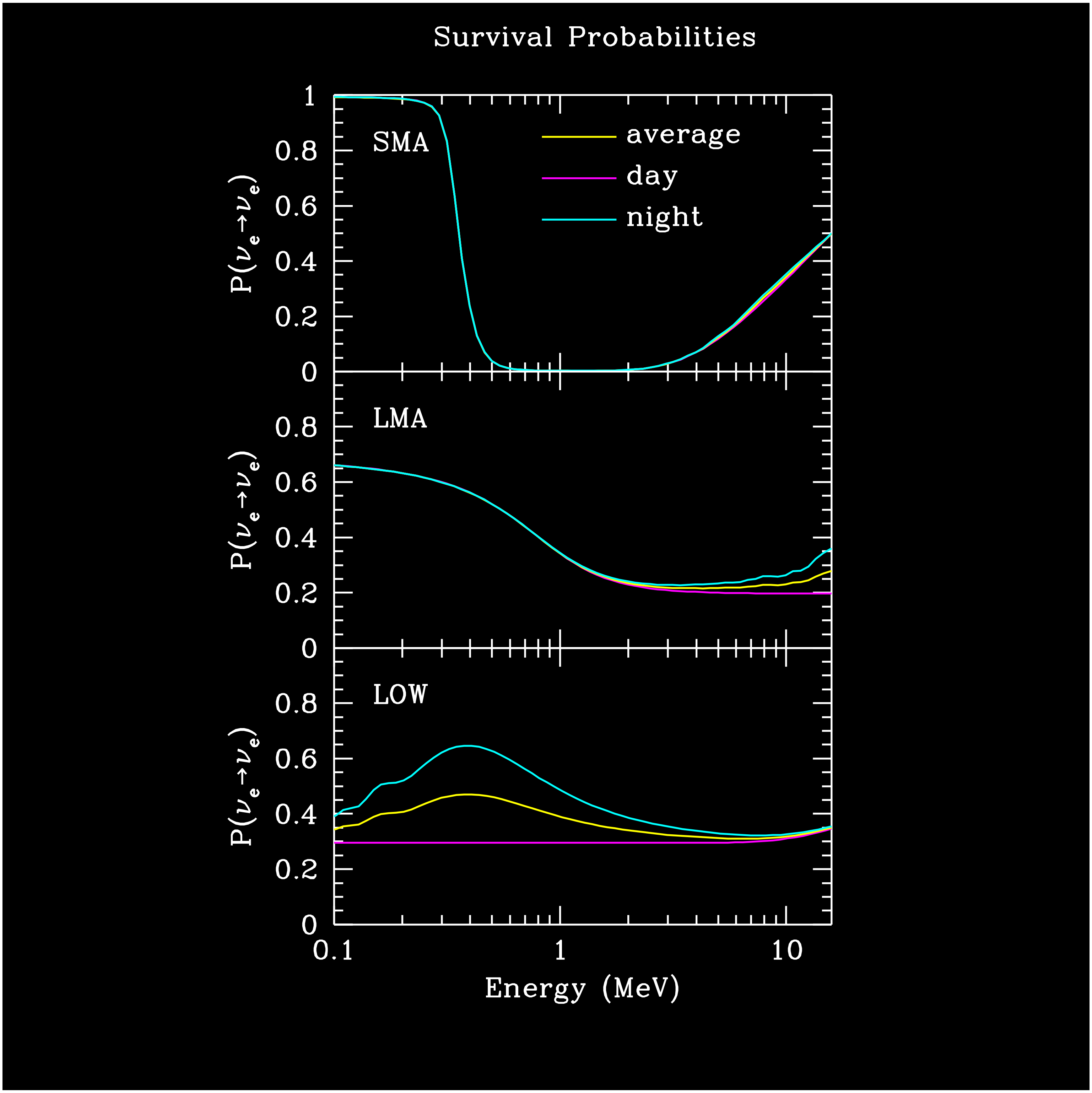
ρ varies in the sun, and the following phenomena might be happening:



νe like up
νμ like down

Thought to be the case (note requires $m_e^2 - m_\mu^2 < 0$ (neutrino masses)

$\Rightarrow$ SMA, LMA, LOW solutions



Survival Probabilities
SMA
LMA
LOW
average
day
night
$P(\nu_e \to \nu_e)$
Energy (MeV)
0.1
1
10
0
0.2
0.4
0.6
0.8
1