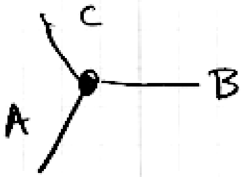
ABC Theory (Simple Feynman Rules)

ABC = 3 distinct particles, $A=\bar{A}$, $B=\bar{B}$, $C=\bar{C}$
m_A m_B m_C

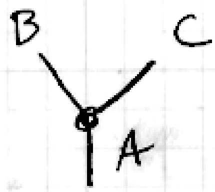


$-ig$

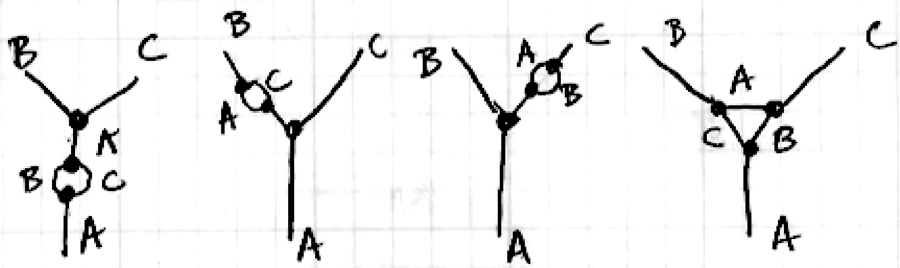
0. Write down all diagrams that can describe a specified process. Will add all amplitudes that are indistinguishable before squaring to get rates. Can approximate by retaining only diagrams with smallest # of vertices.

$A \to BC$

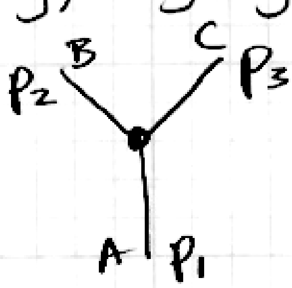
lowest



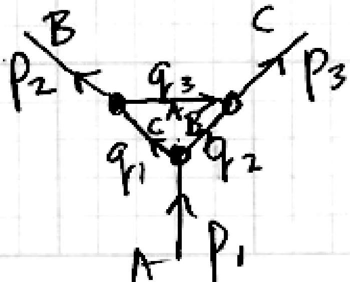
higher



1. Label incoming, outgoing momenta $p_1, p_2, \ldots$



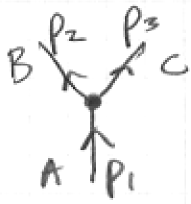
Internal momenta: label q_1, q_2, q_3



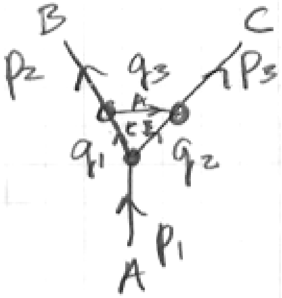
2. factor of $-ig$ for each vertex

(3.) Each internal line: a factor of $\frac{i}{q_j^2 - m_j^2 c^2}$

$$q_j^2 = q_j \cdot q_j = q_j^{0\,2} - |\vec{q}_j|^2$$

so: 

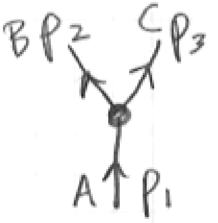
so far ig (no internal momenta)



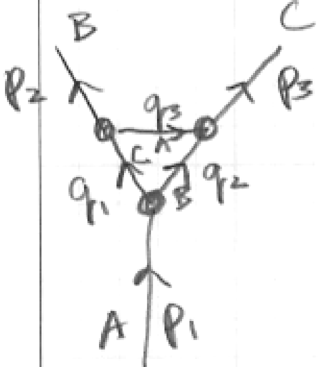
so far $(-ig)^3 \frac{i}{q_1^2 - m_C^2 c^2} \frac{i}{q_2^2 - m_B^2 c^2} \frac{i}{q_3^2 - m_c^2 c^2}$

note: $q_1^2 \neq m_C^2$ for internal lines because this is <u>tunnelling</u>

(4.) Each vertex: $(2\pi)^4 \delta^4 (\underbrace{k_1 + k_2 + k_3}_{\text{sum of momenta INTO vertex}})$



$$-ig (2\pi)^4 \delta^4 (p_1 - p_2 - p_3)$$



so far

$$\frac{g^3 (2\pi)^4 \delta^4 (p_1 - q_1 - q_2)(2\pi)^4 \delta^4 (q_1 - p_2 - q_3)(2\pi)^4 \delta^4 (q_2 + q_3 - p_3)}{[q_1^2 - (m_C c)^2][q_2^2 - (m_B c)^2][q_3^2 - (m_A c)^2]}$$

(5) Integrate over all internal momenta, with factor $\frac{1}{(2\pi)^4} d^4 q_j$

No effect on 

on the "triangle",

$\frac{1}{(2\pi)^4}\int d^4q_1$ puts $q_1 = p_1 - q_2$ resulting in

$$\iint \frac{g^3 (2\pi)^4 \delta^4(p_1 - q_2 - p_2 - q_3)\,\delta^4(q_2 + q_3 - p_3)}{[(p_1-q_2)^2 - (m_C c)^2][q_2^2 - (m_B c)^2][q_3^2 - (m_A c)^2]} \frac{d^4q_2}{(2\pi)^4}\frac{d^4q_3}{(2\pi)^4}$$

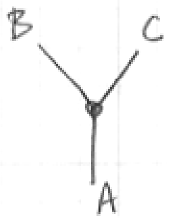
first δ^4, when integrated over q_2,
sets $q_2 = p_1 - p_2 - q_3$
second δ^4 argument becomes $p_1 - p_2 - \cancel{q_3} + \cancel{q_3} - p_3$
$= p_1 - p_2 - p_3$

so, the amplitude for the triangle, so-far, is:

$$= (2\pi)^4 \delta^4(p_1 - p_2 - p_3)\, g^3 \left\{ \int \frac{d^4q_3}{(2\pi)^4} \frac{1}{[(p_2+q_3)^2 - (m_C c)^2][(q_3 - p_1 + p_2)^2 - (m_B c)^2][q_3^2 - (m_A c)^2]} \right\}$$

(6) Cancel out the $(2\pi)^4\delta^4(\Sigma p_i - \Sigma p_f)$; $-i\mathcal{M}$ results.

p_i: initial, p_f: final



$$\mathcal{M}_1 = g$$

$$\mathcal{M}_3 = +ig^3 \int \frac{d^4q_3}{(2\pi)^4} \frac{1}{[(p_2+q_3)^2 - (m_C c)^2][(q_3 - p_1 + p_2)^2 - (m_B c)^2][q_3^2 - (m_A c)^2]}$$

a do-able integral; won't do, but note as $q_3 \to \infty$

$\propto \int \frac{d^4q_3}{q_3^6}$ which is finite.

Lifetime to order g^2:

$$\Gamma = \frac{g^2 |\vec{p}|}{8\pi \hbar m_A^2 c} \qquad |\vec{p}|^2 = \frac{\lambda(m_A^2, m_B^2, m_C^2)\, c^2}{4 m_A^2}$$

note: $|\mathcal{M}_1 + \mathcal{M}_3|^2 = g^2 + \mu_1 g^3 + \mu_2 g^4$

interference generally (Here ??)

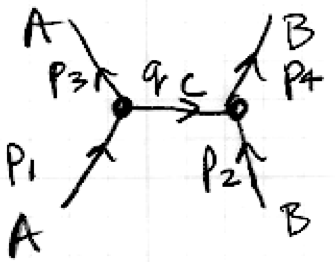
<u>Scattering</u> $AB \to AB$

T-channel

$(2\pi)^4 \delta^4(p_1+p_2-p_3-p_4)$

$\Uparrow$

$q = p_1 - p_3 \Rightarrow \quad p_2 + p_1 - p_3 - p_4$



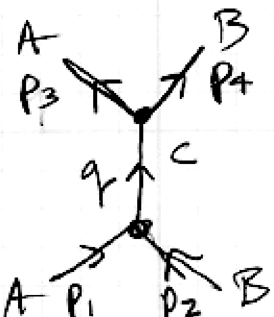
$$(-ig)^2 \int \frac{d^4 q}{(2\pi)^4} \frac{(2\pi)^4 \delta^4(p_1 - p_3 - q)(2\pi)^4 \delta^4(p_2 + q - p_4)\, i}{q^2 - (m_C c)^2}$$

$$\mathcal{M}_t = \frac{g^2}{(p_1 - p_3)^2 - (m_C c)^2}$$

(note, in c-m frame, $E_3 = E_1$ so, q = spacelike)

S-channel

$q = p_1 + p_2 \qquad p_1 + p_2 - p_3 - p_4 \Rightarrow \delta^4(p_1+p_2-p_3-p_4)$



$$(-ig)^2 \int \frac{d^4 q}{(2\pi)^4} \frac{i\,(2\pi)^4 \delta^4(p_1 + p_2 - q)(2\pi)^4 \delta^4(q - p_3 - p_4)}{q^2 - (m_C c)^2}$$

(c-m frame $\vec{p}_1 = -\vec{p}_2$, q = timelike)

$$\mathcal{M}_s = \frac{g^2}{(p_1 + p_2)^2 - (m_C c)^2}$$

$$(p_1 - p_3)^2 = p_1^2 + p_3^2 - 2 p_1 \cdot p_3 = 2 m_A^2 c^2 - 2\Big(\underbrace{\frac{E_1 E_3}{c^2}}_{E_1^2} - |\vec{p}|^2 \cos\theta\Big)$$

center of mass

same c-o-m momentum before, after

$$2 m_A^2 c^2 - 2 E_1^2 / c^2 = -|\vec{p}|^2$$

$$= -|\vec{p}|^2 (1 - \cos\theta) = -2 |\vec{p}|^2 \sin^2 \theta/2$$

$(p_1 + p_2)^2 = s$

$$|\mathcal{M}|^2 = |\mathcal{M}_s + \mathcal{M}_t|^2 = |\mathcal{M}_s|^2 + |\mathcal{M}_t|^2 + \underbrace{2\mathrm{Re}[\mathcal{M}_s^* \mathcal{M}_t]}_{\text{interference term}}$$

$$|\mathcal{M}|^2 = g^4 \left| \frac{1}{-2|\vec{p}|^2 \sin^2\theta/2 - (m_c c)^2} + \frac{1}{s - (m_c c)^2} \right|^2$$

- critical role of $(m_c c)^2 \rightarrow$ terms can change sign around it. In this case, t-channel always negative, but s-channel flips sign.

- when $s = (m_c c)^2$, higher order terms, neglected here, prevent ∞

- here, $|\vec{p}_f| = |\vec{p}_i|$ (elastic)
  $E_1 + E_2 = \sqrt{s}$

$$\frac{d\sigma}{d\Omega} = \left(\frac{\hbar c}{8\pi}\right)^2 \frac{g^4}{s} \times \left| \frac{-1}{2|\vec{p}|^2 \sin^2\frac{\theta}{2} + (m_c c)^2} + \frac{1}{s - (m_c c)^2} \right|^2$$