Masses of L=0 (orbital) Mesons

recall:

$S=0 \quad \langle \vec{S}_1 \cdot \vec{S}_2 \rangle = -\frac{3}{4}\hbar^2$ → singlet (1)

$S=1 \quad \langle \vec{S}_1 \cdot \vec{S}_2 \rangle = +\frac{1}{4}\hbar^2$ → triplet (3)

mnemonic

$1 \times -\frac{3}{4}\hbar^2 + 3 \times \frac{1}{4}\hbar^2 = 0$

"traceless"

For L=0 mesons, we phenomenologically describe masses with the formula

$$M(\text{meson}) = m_1 + m_2 + A \frac{\langle \vec{S}_1 \cdot \vec{S}_2 \rangle}{m_1\, m_2} \qquad \left(\begin{matrix}\text{page } 172 \\ 5.99\end{matrix}\right)$$

the masses m_1, m_2 are in some sense the quark masses, only now we include the extra rest energy of the gluon field. So, m_1 + m_2 will not be bare masses (see inleaf of text) instead; they will be "effective masses"

$m_u = m_d = 310\ \text{MeV}/c^2$ (not 4.2, 7.5 MeV/c^2)

$m_s = 483\ \text{MeV}/c^2$ (not 150 MeV/c^2)

$m_c = 1500\ \text{MeV}/c^2$ (not 1100 MeV/c^2)

$m_b = 4700\ \text{MeV}/c^2$ (not 4200 MeV/c^2)

$m_t = 174{,}000\ \text{MeV}/c^2$

$$A = \left(\frac{1}{\hbar/2} m_u\right)^2 \times 160\ \text{MeV}/c^2$$

$\pi^\pm$, π^0: $M(\pi) = 310 + 310 + \left(\frac{1}{\hbar/2} m_u\right)^2 \times 160 \times \left(-\frac{3}{4}\hbar^2\right)\frac{1}{m_u^2}\ \frac{\text{MeV}}{c^2}$

S=0 $\boxed{M(\pi) = 620 - 480 = 140\ \text{MeV}/c^2 \quad \simeq 139\ \text{MeV}/c^2 \text{ Measured}}$

$K^\pm, \overset{(-)}{K}{}^0$: $S=0$

$$\boxed{\begin{aligned} M(K) &= 483 + 310 + \left(\tfrac{1}{\sqrt{2}}m_u\right)^2 \times 160\left(-\tfrac{3}{4}\hbar^2\right)\frac{1}{m_u m_s} \quad \frac{MeV}{c^2} \\ &= 793 - 480 \times \frac{310}{483} = 485\ MeV/c^2 \\ &\qquad (496\ MeV/c^2) \text{ measured} \end{aligned}}$$

$S=1$:

$\rho^\pm, \rho^0$ $\quad M(\rho) \doteq 310 + 310 + \underline{1} \times 160$

$$\boxed{M(\rho) \doteq 780\ MeV/c^2 \quad (776\ MeV/c^2 \text{ measured})}$$

$K^{*\pm}, \overset{(-)}{K}{}^{*0}$: $\boxed{M(K^*) \doteq 793 + 160 \times \frac{310}{483} = 896\ MeV/c^2 \quad (892\ MeV/c^2) \text{ measured}}$

→ you do rest on homework
(5.22 on p. 188)

→ peculiarity of the η, η' mesons.
Remember: in absence of $s\bar{s}$ quarks,
$\eta = \frac{1}{\sqrt{2}}[|u\bar{u}\rangle + |d\bar{d}\rangle]$. However, the η is
massive enough that annihilation into $s\bar{s}$
disturbs it IF you make the assumption
that $s=d=u$ (neglecting electromagnetism)
then the annihilations give a hamiltonian
that looks like:

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix} u\bar{u} \\ d\bar{d} \\ s\bar{s} \end{pmatrix} = A\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} u\bar{u} \\ d\bar{d} \\ s\bar{s} \end{pmatrix}$$

eigenstates:

$\frac{1}{\sqrt{3}}[|u\bar{u}\rangle + |d\bar{d}\rangle + |s\bar{s}\rangle]$ η'

$\frac{1}{\sqrt{2}}[|u\bar{u}\rangle - |d\bar{d}\rangle]$ π^0

$\frac{1}{\sqrt{6}}[|u\bar{u}\rangle + |d\bar{d}\rangle - 2|s\bar{s}\rangle]$ η

truth is, EVEN THIS IS WRONG, but,

A is closer than ignoring the $s\bar{s}$ entirely (this approximation is called "SU(3) symmetry")

In this approximation,

$$m_\eta = 2\times\frac{1}{6}\, m_{\pi^0} + \frac{4}{6}\times m(s\bar{s}, S=0)$$

$$m(s\bar{s}, S=0) = 483 + 483 - 160\times 3\times\left(\frac{310}{483}\right)^2$$
$$= 768 \text{ MeV}/c^2$$

$$\boxed{m_\eta = \frac{1}{3}\times 140 + \frac{2}{3}\times 768 = 559 \text{ MeV}/c^2}$$

YOU DO $m_{\eta'}$, and get the "surprise"!

<u>Baryons</u> (You are made of baryons)

complicated because of:

1) <u>3</u> spin $\frac{1}{2}$
2) color is important (getting the qq attraction!)

$$\Psi_{baryon} = \Psi(\text{space})\Psi(\text{spin})\Psi(\text{flavor})\Psi(\text{color}) \quad \left(\begin{matrix}5.109\\ p.176\end{matrix}\right)$$

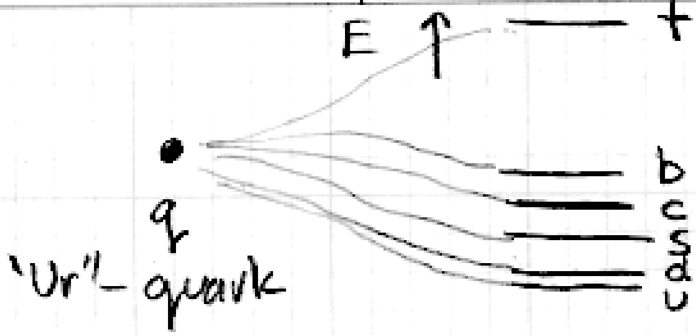
→ talked about this; antisymmetric under exchange of any pair

pair flipped

$$|\Psi(\text{color})\rangle = \frac{1}{\sqrt{6}}\left[|RGB\rangle - |RBG\rangle + |GBR\rangle - |GRB\rangle + |BRG\rangle - |BGR\rangle\right] \quad \left(\begin{matrix}p.178\\ 5.111\end{matrix}\right)$$

quark #1 red — quark #2 green — quark #3 blue

To think about the rest, need to generalize the pauli principal. Think of the 6 quarks as "excitations" of the "fundamental" quark



"flavor" quantum #

since $m_u \simeq m_d$ some odd things happen

The Pauli principal says:

$$\underline{\text{Exchange}}\begin{pmatrix}1 \text{ quark} \\ 1 \text{ other quark}\end{pmatrix}|\Psi_{baryon}\rangle = -|\Psi_{baryon}\rangle$$

but we already know

$$\underset{\sim}{E}|\Psi(\text{color})\rangle = -|\Psi(\text{color})\rangle$$

also, for the <u>GROUND</u> state, all 3 quarks are in (relative) $L=0$ states.

$$\underset{\sim}{E}|\Psi(\text{space})\rangle = +|\Psi(\text{space})\rangle$$

This means that the remaining parts of the state must be symmetric under exchange:

$$\boxed{\underset{\sim}{E}|\Psi(\text{spin})\Psi(\text{flavor})\rangle = +|\Psi(\text{spin})\Psi(\text{flavor})\rangle}$$

$\Psi(\text{spin})$ & $\Psi(\text{flavor})$ can be both <u>symmetric</u> under exchange (Baryon "decuplet")

spin $\rightarrow$ $|\uparrow\uparrow\uparrow\rangle$ "obviously" symmetric

z component: $+\frac{3}{2}\hbar$ angular momentum

$S = 3/2$

flavor $\rightarrow$ $|uuu\rangle$

$\frac{1}{\sqrt{3}}(|uud\rangle + |udu\rangle + |duu\rangle)$

$\frac{1}{\sqrt{3}}(|ddu\rangle + |dud\rangle + |udd\rangle)$

$|ddd\rangle$

or use any flavor

This type of baryon has spin-3/2; high mass.

But protons have spin-$\frac{1}{2}$, how does that work?

$\psi(\text{spin}) \rightarrow$ antisymmetric under exchange

$\psi(\text{flavor}) \rightarrow$ antisymmetric under exchange.

This gets complicated!!

Flavor: $\frac{1}{\sqrt{2}}[|uu\rangle - |uu\rangle] = 0$!

Consequence: no spin-$\frac{1}{2}$ uuu sss ddd ccc states

When 2 quarks have different flavors the trick is:

say:

$$\underbrace{\frac{1}{\sqrt{2}}[\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow]}_{\text{spin part}} \times \underbrace{\frac{1}{\sqrt{2}}[udu - duu]}_{\text{flavor part}}$$

means #1#2#3

But then: why doesn't d get to be in the #3 slot? Griffiths goes into this. p.179

the above is: $\psi_{12}(\text{spin})\psi_{12}(\text{flavor})$

the proton wave function is then.

$$= \frac{\sqrt{2}}{3}\left[\psi_{12}(\text{spin})\psi_{12}(\text{flavor}) + \psi_{23}(\text{spin})\psi_{23}(\text{flavor}) + \psi_{13}(\text{spin})\psi_{13}(\text{flavor})\right].$$

$$= \frac{1}{\sqrt{18}} \Big[2|u(\uparrow)u(\uparrow)d(\downarrow)\rangle - |u(\uparrow)u(\downarrow)d(\uparrow)\rangle - |u(\downarrow)u(\uparrow)d(\uparrow)\rangle + \text{2 other permutations} \Big]$$

with d in other places.

Let's now use this to get the magnetic moment of the proton: (can ignore the permutations)

$$\mu_u = \cancel{g} \cdot \frac{2 \cdot 1}{2m_u c} \times \frac{\hbar}{2} \times +\frac{2}{3}e = \frac{2}{3}\frac{e\hbar}{2m_u c}$$

$$\mu_d = -\frac{1}{3}\frac{e\hbar}{2m_d c} \qquad m_u \simeq m_d$$

permutations

$$\Rightarrow \frac{3}{18}\Big[4\times(2\mu_u - \mu_d) + \mu_d + \mu_d\Big]$$

$$\mu_p = \frac{24}{18}\mu_u - \frac{6}{18}\mu_d = \frac{4}{3}\mu_u - \frac{1}{3}\mu_d$$

$$= \left(\frac{8}{9} + \frac{1}{9}\right)\frac{e\hbar}{2m_u c} = \frac{m_p}{m_u}\frac{e\hbar}{2m_p c}$$

$\simeq 3$; more carefully, $= 2.79$, measure 2.793

<u>neutron</u>:

$$\frac{4}{3}\mu_d - \frac{1}{3}\mu_u$$

$$\mu_n = \left(-\frac{4}{9} - \frac{2}{9}\right)\frac{e\hbar}{2m_u c} = -\frac{2}{3}\frac{m_p}{m_u}\frac{e\hbar}{2m_p c}$$

$\simeq -2$; more carefully, $= -1.86$

but: $\frac{\mu_n}{\mu_p} = -\frac{2}{3}$ is very nearly true

5.118, p 182

Continuing: $|uds\rangle$ baryons more subtle;

ends up, $\underline{2}$ distinct ways of making a symmetric spin×flavor state.

$\Rightarrow$ all magnetic moments of baryons in reasonable agreement.