Physics 115C Third Problem Set

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1. (a) Start from the Lagrangian in one dimension for two particles that interact through a potential:

   \[ L = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2 - V(x_1 - x_2). \]

   Change position variables to \( X \) and \( x \) where

   \[ X = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}, \quad x = x_1 - x_2 \]

   and determine the momenta, \( P \) and \( p \), that are conjugate to \( X \) and \( x \), by evaluating \( \partial L/\partial X \) and \( \partial L/\partial \dot{x} \). Then express \( P \) and \( p \) in terms of \( p_1 \) and \( p_2 \), the momenta conjugate to \( x_1 \) and \( x_2 \).

   (b) Now treat coordinate and momentum variables as operators. Assume that \([x_1, p_1] = [x_2, p_2] = i\hbar\), and explicitly evaluate \([X, P], [X, p], [x, P], \) and \([x, p]\) using the expressions from part (a).

2. Imagine the hydrogenic atom that consists of a negative \( \tau \) lepton, which has charge \(-1\) and \( m_\tau c^2 = 1777 \) MeV, bound to a proton. Evaluate numerically the binding energy of the ground state, as well as the length scale \( \alpha \) defined on page 20 of the notes.

3. (a) Find the eigenvalues and eigenvectors of the operator that represents \( s_y \):

   \[ \frac{\hbar}{2} \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \]

   (b) Find the values of \( \theta \) and \( \phi \) necessary to express the eigenvectors in part (a) in the form

   \[ \begin{bmatrix} e^{-i\phi/2} \cos \theta/2 \\ e^{i\phi/2} \sin \theta/2 \end{bmatrix} \]

   Don’t worry if there is a common phase factor of difference between this part and your result from part (a).

   (c) Evaluate the expectation value of \( s_y \) for general \( \theta \) and \( \phi \), using the general state vector parameterized in part (b).

4. Explicitly find the eigenvalues and eigenvectors of the matrix:

   \[ \sigma_n = \cos \theta \sigma_x + \sin \theta \cos \phi \sigma_x + \sin \theta \sin \phi \sigma_y \]