Physics 115C Third Problem Set

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1. (a) Start from the Lagrangian in one dimension for two particles that interact through a potential:
$$L = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2 - V(x_1 - x_2).$$
Change position variables to X and x where
$$X = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} \quad , \quad x = x_1 - x_2$$
and determine the momenta, P and p, that are conjugate to X and x, by evaluating $\partial L/\partial \dot{X}$ and $\partial L/\partial \dot{x}$. Then express P and p in terms of p_1 and p_2, the momenta conjugate to x_1 and x_2.

   (b) Now treat coordinate and momentum variables as operators. Assume that $[x_1, p_1] = [x_2, p_2] = i\hbar$, and explicitly evaluate $[X, P]$, $[X, p]$, $[x, P]$, and $[x, p]$ using the expressions from part (a).

2. Imagine the hygrogenic atom that consists of a negative τ lepton, which has charge -1 and $m_\tau c^2 = 1777\,\mathrm{MeV}$, bound to a proton. Evaluate numerically the binding energy of the ground state, as well as the length scale $\tilde{a}$ defined on page 20 of the notes.

3. (a) Find the eigenvalues and eigenvectors of the operator that represents s_y:
$$\frac{\hbar}{2} \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

   (b) Find the values of θ and ϕ necessary to express the eigenvectors in part (a) in the form
$$\begin{bmatrix} e^{-i\phi/2} \cos\theta/2 \\ e^{i\phi/2} \sin\theta/2 \end{bmatrix}$$
Don't worry if there is a common phase factor of difference between this part and your result from part (a).

   (c) Evaluate the expectation value of s_y for general θ and ϕ, using the general state vector parameterized in part (b).

4. Explicitly find the eigenvalues and eigenvectors of the matrix:
$$\sigma_n = \cos\theta\,\sigma_z + \sin\theta\cos\phi\,\sigma_x + \sin\theta\sin\phi\,\sigma_y$$