

## "Feynman" Diagram

$$\begin{aligned} \tilde{U}_I(t, t_0) &= 1 - \frac{i}{\hbar} \int_{t_0}^t \tilde{H}'_I(t') dt' \\ &\quad + \left(-\frac{i}{\hbar}\right)^2 \int_{t_0}^t \int_{t_0}^{t'} \tilde{H}'_I(t') \tilde{H}'_I(t'') dt' dt'' + \dots \end{aligned}$$

⇒ back to Schrödinger Picture

$$|\Psi_I(t)\rangle = \tilde{U}_I(t, t_0) |\Psi_I(t_0)\rangle$$

$$\tilde{U}_S^{t_0}(t, t_0) |\Psi_S(t)\rangle = \tilde{U}_I(t, t_0) \tilde{U}_S^{o t}(t_0, t_0) |\Psi_S(t_0)\rangle$$

$$\downarrow \qquad \qquad \qquad \downarrow$$

$$\tilde{U}_S^{t_0}(t, t_0) \tilde{U}_S(t, t_0) |\Psi_S(t_0)\rangle = \tilde{U}_I(t, t_0) |\Psi_S(t_0)\rangle$$

$$\boxed{\tilde{U}_S(t, t_0) = \tilde{U}_S^{o t}(t, t_0) \tilde{U}_I(t, t_0)}$$

$$\tilde{H}'_I(t') = \tilde{U}_S^{o t}(t', t_0) \tilde{H}'_S(t') \tilde{U}_S^o(t', t_0)$$

so,

$$\tilde{U}_S^{o t}(t, t_0) \tilde{U}_I(t, t_0) = \tilde{U}_S(t, t_0) = \tilde{U}_S^o(t, t_0)$$

$$- \frac{i}{\hbar} \int_{t_0}^t \underbrace{\tilde{U}_S^{o t}(t, t_0) \tilde{U}_S^{o t}(t', t_0)}_{\text{study}} \tilde{H}'_S(t') \tilde{U}_S^o(t', t_0)$$

+ higher order

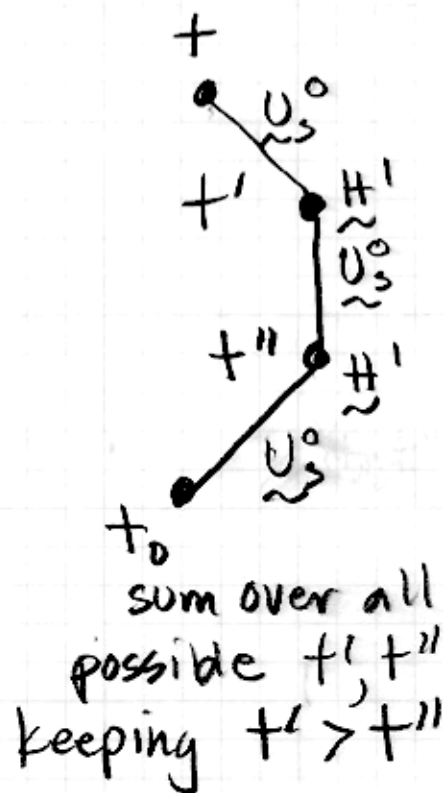
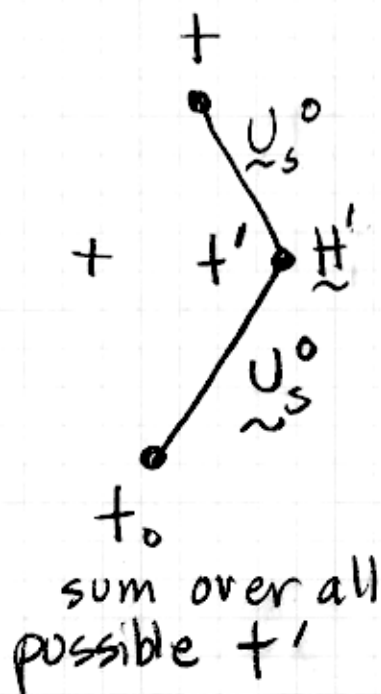
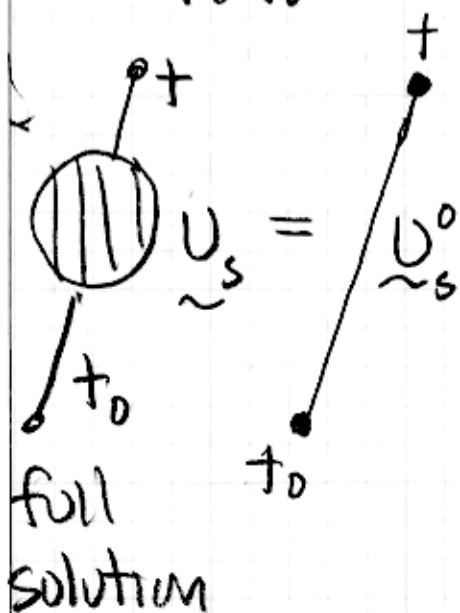
$$\begin{aligned} \underset{\sim}{U}_s^0(t, t_0) \underset{\sim}{U}_s^0{}^\dagger(t', t_0) &= \underset{\sim}{U}_s^0(t, t_0) \underset{\sim}{U}_s^0(t_0, t') \\ &= \underset{\sim}{U}_s^0(t, t'). \end{aligned}$$

similar "composition" influences second & higher order:

$$\underset{\sim}{U}_s(t, t_0) = \underset{\sim}{U}_s^0(t, t_0) - \frac{i}{\hbar} \int_{t_0}^t \underset{\sim}{U}_s^0(t, t') \underset{\sim}{H}'(t') \underset{\sim}{U}_s^0(t', t_0) dt'$$

$$+ \left(\frac{-i}{\hbar}\right)^2 \int_{t_0}^t \int_{t_0}^{t'} \underset{\sim}{U}_s^0(t, t') \underset{\sim}{H}'(t') \underset{\sim}{U}_s^0(t', t'') \underset{\sim}{H}'(t'') \underset{\sim}{U}_s^0(t'', t_0) dt'' dt'$$

← read right to left



Material Since Midterm (approximately)

A. Spin 14.1, 14.2

14.3 1) Commutation Relations

2) "2-component" formalism

3) Spin operators - representation of a 3-vector in the 2-component formalism, meaning of  $\hat{n} \cdot \vec{S}$  (14.3.27).

4) Pauli Matrices & Pauli Gymnastics

5) Rotation Operators

14.4 6) Spin Dynamics - torque, energy

7) precession

8)  $\gamma = \frac{gq}{2mc}$ ,  $\vec{\omega}_0 = -\gamma \vec{B}$ ,  $\vec{\mu} = \gamma \vec{S}$ ,

$$\underline{H} = -\vec{\mu} \cdot \vec{B} = -\gamma \vec{S} \cdot \vec{B} = -\frac{1}{2} \hbar \vec{\omega}_0 \cdot \vec{\sigma}$$

$$g = 2$$

9) Paramagnetic Resonance

14.5 10) Stern Gerlach Experiments

a) General Functioning

b) Abstract Implementation of Postulate III (notes: 51-54)

c) multiple Stern Gerlachs.

## B. Addition of Angular Momenta

- 15.1
- 1) Concept of product ket
  - 2) That  $\vec{J}_1 \cdot \vec{J}_2$  not diagonal in product basis, so  $(\vec{J}_1 + \vec{J}_2)^2 = \vec{J}^2 = \vec{J}_1^2 + \vec{J}_2^2 + 2\vec{J}_1 \cdot \vec{J}_2$  not diagonal;  $\vec{J}_1^2, \vec{J}_2^2$  are diagonal
- 15.2
- 3) How  $J_1 - J_2 \leq J \leq |J_1 + J_2|$  preserves the # of states.
  - 4) How to start from  $|j_1 m_1 j_2 m_2\rangle = |(j_1 + j_2)(j_1 + j_2), j_1 j_2\rangle$  and lower to get Clebsch-Gordan Coefficients.
  - 5) How to skip to  $|(j_1 + j_2 - 1)(j_1 + j_2 - 1), j_1 j_2\rangle$  by orthogonality.
  - 6) Notation:  $|j_1 m_1 j_2 m_2\rangle, |j_1 m_1 j_2 m_2\rangle$   

$$\langle j_1 m_1 j_2 m_2 | j_1 m_1 j_2 m_2 \rangle = \langle j_1 m_1 j_2 m_2 | j_1 m \rangle$$

even =  $\langle m_1 m_2 | j_1 m \rangle$
  - 7) That  $m_1 + m_2 = m$
  - 8) Modified Spectroscopic Notation  
 $\Rightarrow$  nothing from 15.3, 15.4, Chapter 16

## C. Time-Independent Perturbation Theory

17.1 1)  $\underline{H} = \underline{H}^0 + \underline{H}'$ , crude:  $|\underline{H}'| \ll |\underline{H}^0|$

$$|n\rangle = |n^0\rangle + |n^1\rangle + |n^2\rangle + \dots$$

$$E_n = E_n^0 + E_n^1 + E_n^2 + \dots$$

→ Non-degenerate.

2)  $E_n^1 = \langle n^0 | \underline{H}' | n^0 \rangle$  17.1.7

3) To first order

$$|n\rangle = |n^0\rangle e^{i\alpha} + \sum_m \frac{|m^0\rangle \langle m^0 | \underline{H}' | n^0 \rangle}{E_n^0 - E_m^0}$$

17.1.3

4)  $E_n^2 = \sum_m \frac{|\langle n^0 | \underline{H}' | m^0 \rangle|^2}{E_n^0 - E_m^0}$

17.2 5) Examples from S.H.O, Hydrogen Atom, Square Well... maybe more.

6) Selection Rules:

a)  $[\underline{L}, \underline{H}'] = 0 \Rightarrow$  perturbation diagonal in eigenkets of  $\underline{L}$ .

b) Parity... when  $l' = l \pm 1, l \pm 3, \dots$   
and when  $l' = l, l \pm 2, \dots$

7) Dalgarno Lewis p. 462

17.3 8) Degenerate, First Order (Diagonalize in Degenerate Subspace)  
a) Stark Effect.

## D. Time Dependent Perturbation Theory

$$1) \tilde{H} = \tilde{H}^0 + \tilde{H}'(t)$$

↑  
static

↑ small, time dependent

$$2) d_f(t) = \delta_{fi} - \frac{i}{\hbar} \int_0^t \langle f^0 | \tilde{H}'(t') | i^0 \rangle e^{i\omega_{fi}t'} dt'$$

first order

### 3) Sudden versus Adiabatic

↑ - time scale variation of  $\tilde{H}'(t)$

$$\omega_{\min} = \text{minimum}(\omega_{fi})$$

$\omega_{\min} \tau \gg 1$  Adiabatic  $\rightarrow$  "states track"

$\omega_{\min} \tau \ll 1$  Sudden  $\rightarrow$  transitions

### 4) Fermi's Golden Rule

5) "Pictures" - Schrödinger  
Heisenberg

Interaction (Dirac).

## Material Prior to Midterm

### A. Radial Equation.

1) know  $Y_{\ell}^m$ 's (pp. 335-339)

12.6 2) Radial Equation

a)  $R_{E\ell} = U_{E\ell} / r$

b)  $U_{E\ell} \propto r^{2\ell+1}$  at origin

pp. 346  $\Rightarrow$  352 not covered.

c) fall to the center

d) finite/ $\infty$  # of bound

states:  $r^2 V(r) \rightarrow \infty$  as  $r \rightarrow \infty$   
 $\infty$  bound states

$\rightarrow$  constant or 0  
 finite #

e) Power Law Potentials

$\rightarrow$  length scale

$\rightarrow$  energy scale

$\rightarrow$  "reduced" equation

3) Numerical Integration Technique

### B. Hydrogenic Atoms

13.1 1) Energy Eigenvalues

2) Angular Momentum, Degeneracy.

3) Radial Wave Functions

4) Powers of  $\alpha = e^2/\hbar c$

$\frac{v}{c} \cong \alpha$  in Hydrogen Ground State.

$$E_n = -\frac{1}{2} \mu c^2 \times \frac{\alpha^2}{n^2} \quad \text{et cetera.}$$

5) Two body problem,  $\mu = \frac{m_1 m_2}{m_1 + m_2}$   
et. cetera.

6) Other Bound States

a) quarkonium

b) deuterons