Change of variable

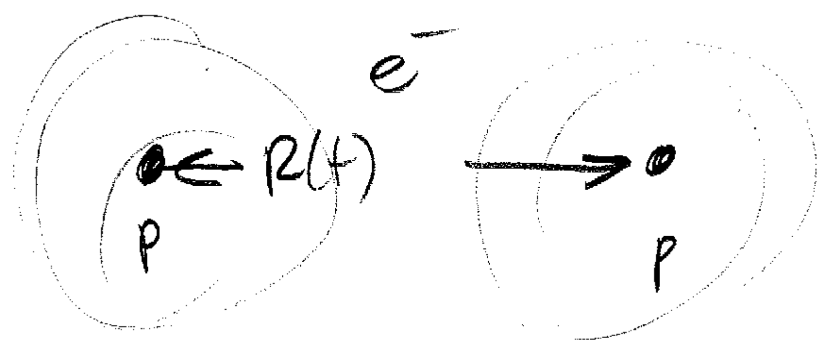
Usually $\psi_n(t') =$ some function of a spatial variable

Example: $\psi_n(t') = \sqrt{\frac{2}{a(t')}} \sin\left(\frac{n\pi x}{a(t')}\right)$

varying square well

a is spatial variable.
(call a $R(t')$)

another



$$\frac{\partial \psi_n}{\partial t'} = \frac{\partial \psi_n}{\partial R} \frac{dR}{dt}$$

$$\gamma_n = i \int_0^t \langle \psi_n(t') | \frac{\partial}{\partial t'} \psi_n(t') \rangle dt'$$

$$\psi_n^* \frac{\partial}{\partial t'} \psi_n$$

$$= i \int_0^t \langle \psi_n(t') | \frac{\partial \psi_n}{\partial R} \rangle \frac{dR}{\cancel{dt'}} \cancel{dt'}$$

$$= i \int_{R(0)}^{R(t)} \langle \psi_n | \frac{\partial \psi_n}{\partial R} \rangle dR$$

sometimes wavefunction described by <u>multiple</u> parameters $(R_1, R_2, \ldots)$

$$\frac{\partial \psi_n}{\partial t'} = \frac{\partial \psi}{\partial R_1}\frac{dR_1}{dt'} + \frac{\partial \psi}{\partial R_2}\frac{dR_2}{dt'} + \cdots$$

$$= \left(\vec{\nabla}_R \psi_n\right) \cdot \frac{d\vec{R}}{dt'}$$

$$\gamma_n = i \underbrace{\int_{\vec{R}_i}^{\vec{R}_f} \langle \psi_n | \vec{\nabla} \psi_n \rangle \cdot d\vec{R}}_{\text{connection}}$$

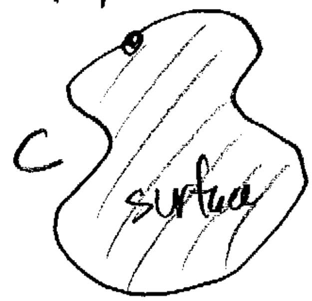
connection... suppose $\vec{R}_i = \vec{R}_f$

$\gamma_n = 0$ when "holonomic"

$\neq 0$ $\underbrace{\text{non-holonomic}}$

in this case...

$\vec{R}_i = \vec{R}_f$

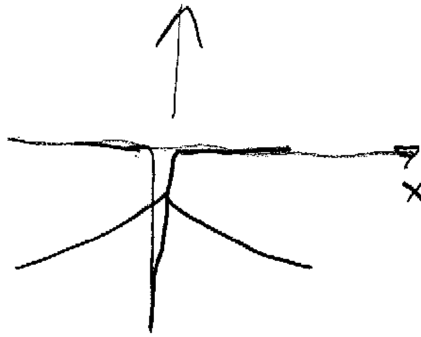


$$\gamma_n = i \oint_C \langle \psi_n | \vec{\nabla} \psi_n \rangle \cdot d\vec{R}$$

$$= i \int_{\text{surface}} \vec{\nabla}_R \times \left[\langle \psi_n | \vec{\nabla} \psi_n \rangle\right] \cdot d\vec{a}$$

$\vec{R}$-space

$\boxed{10.4}$ Bound state in 1-d δ-function



$-\alpha\delta(x)$

$$\psi(x) = Be^{\kappa x} \quad x \le 0$$
$$= Be^{-\kappa x} \quad x \geq 0$$

$\kappa = \frac{m}{\hbar^2}\alpha$ satisfies S.E.

$$E = -\frac{m}{2\hbar^2}\alpha^2$$

norm $\quad \psi(x) = \frac{m^{1/2}\alpha^{1/2}}{\hbar} e^{-\frac{m\alpha|x|}{\hbar^2}}$

α: gradually increases from α_1 to α_2 -- $\frac{d\alpha}{dt} = c$

$$\frac{\partial\psi}{\partial\alpha} = \underbrace{\frac{1}{2\alpha}\psi(x)}_{\text{first term}} - \underbrace{\frac{m|x|}{\hbar^2}\psi(x)}_{\text{second}} \Big] \quad x = 0 ?$$

$$\langle\psi(x)|\frac{\partial\psi}{\partial\alpha}\rangle$$
$$= \underbrace{\frac{1}{2\alpha}\int_{-\infty}^{\infty}|\psi(x)|^2 dx}_{\text{easy} = \frac{1}{2\alpha}} - \frac{m}{\hbar^2}\int_{-\infty}^{\infty} dx\,|x|\,|\psi(x)|^2$$

$$\frac{m}{\hbar^2}\frac{m\alpha}{\hbar^2}\int_{-\infty}^{\infty} dx\,|x|\,e^{-2m\alpha|x|/\hbar^2}$$

$$= \frac{1}{4\alpha}\cdot 2\cdot\int_0^{\infty}\left(\frac{2m\alpha\,dx}{\hbar^2}\right) \times \left(\frac{2m\alpha x}{\hbar^2}\right) \times e^{-2m\alpha x/\hbar^2}$$

$$= \frac{1}{2\alpha}\int_0^{\infty} dz\, z\, e^{-z}$$

$$= \frac{1}{2\alpha}$$

$$\boxed{\langle\psi(x)|\frac{\partial\psi}{\partial\alpha}(x)\rangle = \frac{1}{2\alpha} - \frac{1}{2\alpha} = 0}$$

$$\therefore\ \gamma = 0.$$

NOTICE

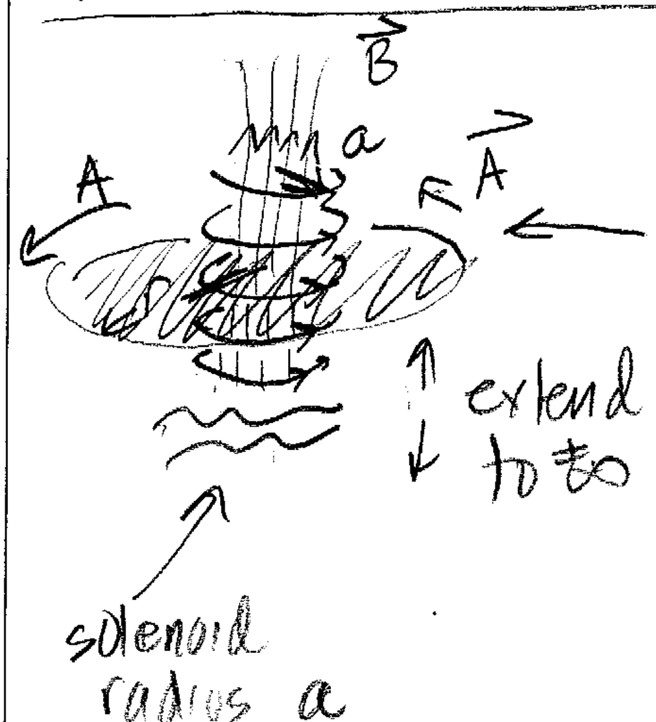
$$\langle\psi_m|\psi_m\rangle = 1$$

$$\langle\frac{\partial\psi_m}{\partial R}|\psi_m\rangle + \langle\psi_m|\frac{\partial\psi_m}{\partial R}\rangle = 0$$

$$2\,\mathrm{Re}\left(\underbrace{\langle\psi_m|\frac{\partial\psi_m}{\partial R}\rangle}\right) = 0$$

if this expression real valued $= 0$

Aharanov - Bohm Effect



$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$\int \vec{B} \cdot d\vec{a} = \pi a^2 B$$

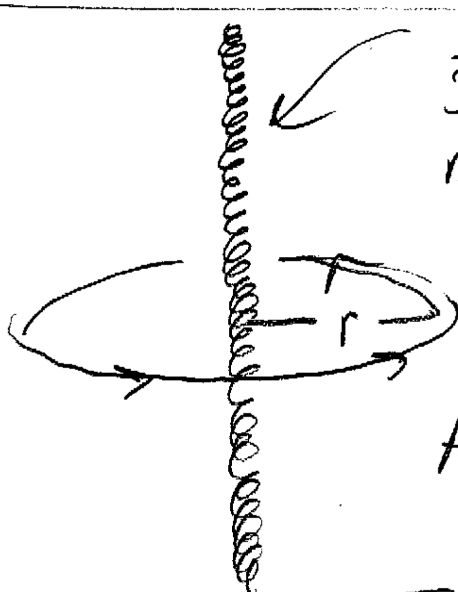
$$= \int (\vec{\nabla} \times \vec{A}) d\vec{a}$$

$$= \oint \vec{A} \cdot d\vec{r}$$

$$= 2\pi r A_\phi$$

$$A_\phi = \frac{(\pi a^2 B)}{2\pi r}$$

$$A_\phi = \frac{a^2}{2}\frac{B}{r}$$



$\vec{B}$ all inside
no $\vec{B}$ outside.

$\vec{A} \neq 0$
outside!

$$A_\phi = \frac{\text{constant}}{R}$$

Is it "real"?

ϕ (electric potential) was "real"

added $q\phi$ to H

$\vec{A}$... subtract $q\vec{A}$ from $\vec{p}$:

$$H = \frac{1}{2m}\Big(\underbrace{\frac{\hbar}{i}\vec{\nabla}}_{\vec{p}} - q\vec{A}\Big)^2 + q\phi$$

Suppose ... ψ' solves SE

without $\vec{A}$ ... $\vec{A} = 0$

$$H\psi' = E\psi' \qquad H = \frac{1}{2m}\left(\frac{\hbar}{i}\vec{\nabla}\right)^2 + q\phi$$

claim: $e^{i\frac{q}{\hbar}\int^{\vec{r}}\vec{A}(\vec{r}\,')\cdot d\vec{r}\,'}\psi'$

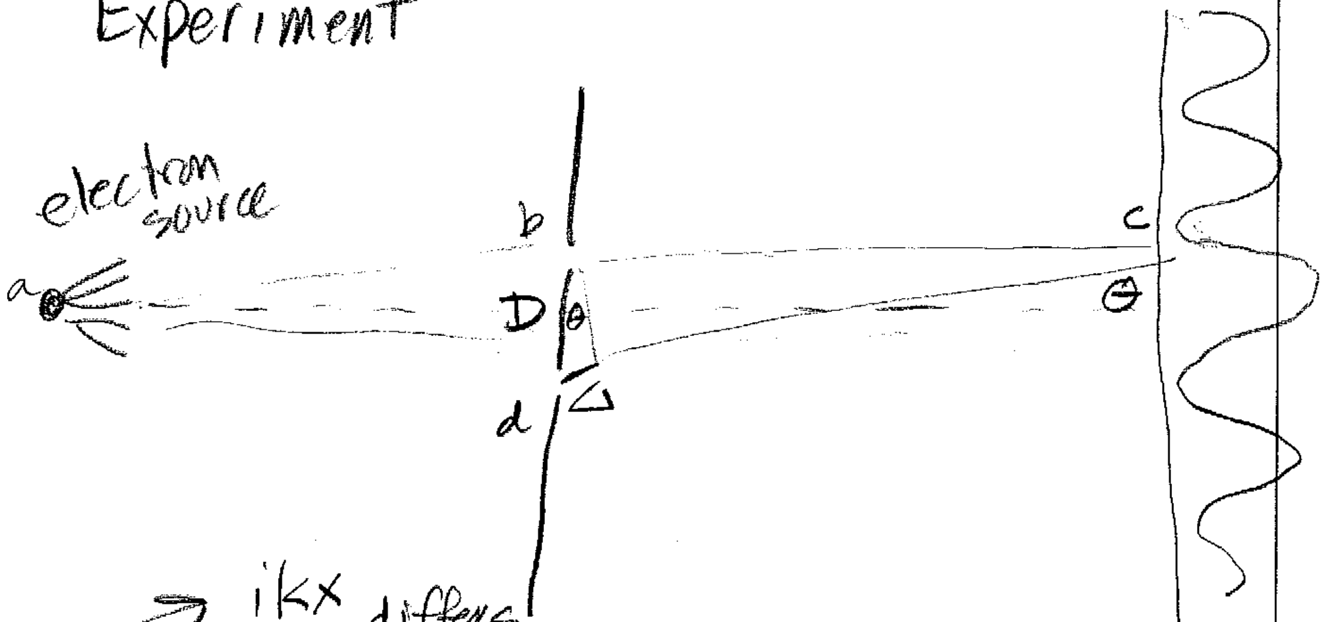
solves equation with $\vec{A} \neq 0$

$$\left(\frac{\hbar}{i}\vec{\nabla} - q\vec{A}\right) e^{\frac{iq}{\hbar}\int_0^{\vec{r}}\vec{A}(\vec{r}\,')\cdot d\vec{r}\,'}\psi'$$

$$= \Big(\frac{\hbar}{i}\vec{\nabla}\,\frac{iq}{\hbar}\int^{\vec{r}}\vec{A}(\vec{r}\,')\cdot d\vec{r}\,' - q\vec{A}\Big)e^{i[\,]}\psi'$$

$$+ e^{i[\,]}\frac{\hbar}{i}\vec{\nabla}\psi'$$

Experiment



e^{ikx} differs

when $\Delta = \text{odd} \times \frac{\lambda}{2}$, minimum.

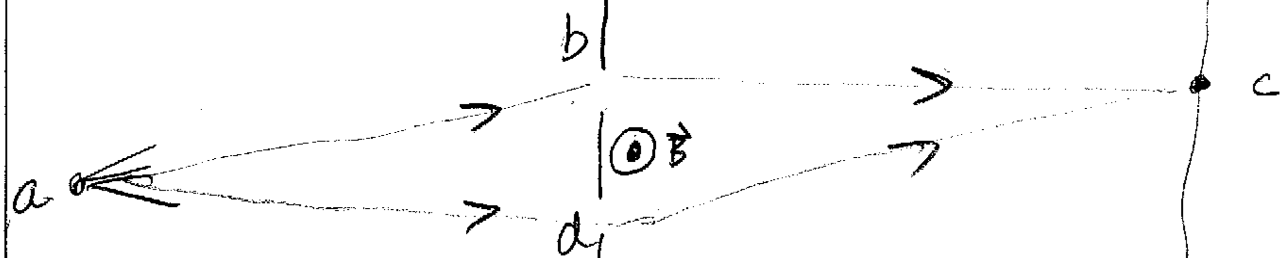
$D\theta = \text{odd} \times \frac{\lambda}{2}$

$\theta = (1, 3, \ldots) \frac{\lambda}{2D}$

$\lambda = \frac{h}{p}$ (De Broglie).

results from

$|\psi'_{abc} + \psi'_{adc}|^2$



$\psi'_{abc} \to e^{i\phi_{abc}} \psi'_{abc}$

$\psi'_{adc} \to e^{i\phi_{adc}} \psi'_{adc}$

$$\phi_{abc} - \phi_{adc} = \frac{q}{\hbar} \oint \vec{A}(\vec{r}\,') \cdot d\vec{r}\,'$$

$$= \frac{q}{\hbar} \Phi_B$$

$$\psi_c = e^{i\phi_{abc}}\psi'_{abc} + e^{i\phi_{adc}}\psi'_{adc}$$

$$= e^{i\phi_{adc}}\left[e^{\frac{iq}{\hbar}\Phi_B}\psi'_{abc} + \psi'_{adc}\right]$$

$$|\psi_c|^2 = \left|e^{\frac{iq}{\hbar}\Phi_B}\psi'_{abc} + \psi'_{adc}\right|^2$$

observable phase!

Aharanov - Bohm

make $\frac{q\phi_B}{\hbar} = \pi$ ... central maximum becomes a minimum!