$$\dot{c}_a e^{\frac{-iE_a t}{\hbar}} = \frac{1}{i\hbar}\left[H'_{aa} c_a e^{\frac{-iE_a t}{\hbar}} + H'_{ab} c_b e^{\frac{-iE_b t}{\hbar}}\right]$$

$$\boxed{\begin{aligned} \dot{c}_a &= -\frac{i}{\hbar}\left[c_a H'_{aa} + c_b H'_{ab} e^{-i(E_b - E_a)t/\hbar}\right] \\ \dot{c}_b &= -\frac{i}{\hbar}\left[c_b H'_{bb} + c_a H'_{ba} e^{i(E_b - E_a)t/\hbar}\right] \\ &\quad 9.10 + 9.11 \quad p. 342 \end{aligned}}$$

$$\omega_0 \equiv \frac{E_b - E_a}{\hbar}$$

Frequently,

$$H'_{aa} = H'_{bb} = 0$$

9.3:

$$H' = \begin{pmatrix} 0 & \alpha \\ \alpha & 0 \end{pmatrix} \delta(t)$$

δ function!

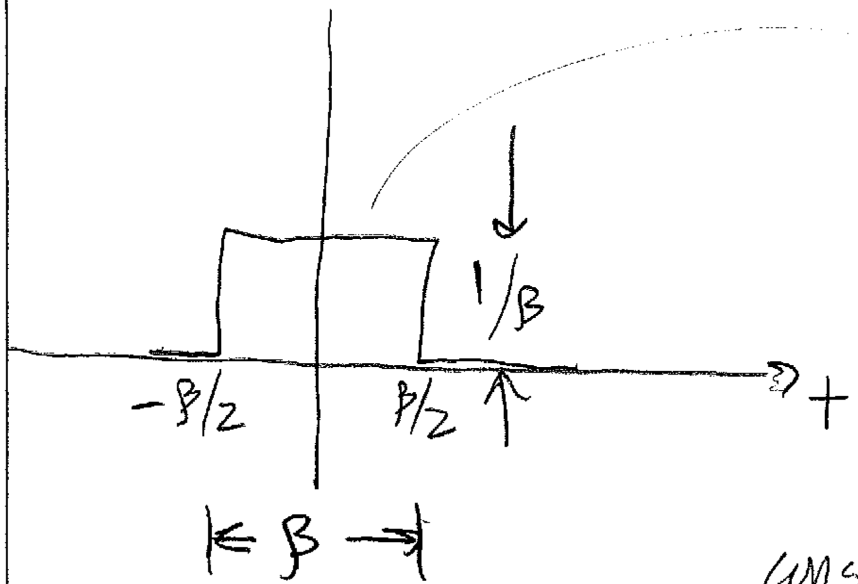
Initial condition:

$$c_a(-\infty) = 1$$
$$c_b(-\infty) = 0$$

$$\dot{c}_a = -\frac{i}{\hbar} c_b \alpha \delta(t)$$

$$\dot{c}_b = -\frac{i}{\hbar} c_a \alpha \delta(t)$$

Approximate δ-function:



key point:

constant in here!

In there:

$$c_a(-\infty) = 1$$
$$c_b(0) = 0$$

$$\dot{c}_a = -\frac{i}{\hbar}\frac{\alpha}{\beta}\, c_b$$

$$\boxed{\dot{c}_b = \frac{-i}{\hbar}\frac{\alpha}{\beta}\, c_a}$$

$$\dot{c}_a = i\hbar\frac{\beta}{\alpha}\,\ddot{c}_b$$

$$i\hbar\frac{\beta}{\alpha}\ddot{c}_b = -\frac{i}{\hbar}\left(\frac{\alpha}{\beta}\right) c_b$$

$$\ddot{c}_b = -\frac{\alpha^2}{\beta^2\hbar^2}\, c_b$$

$$c_b(-\beta/2) = 0$$

$$c_b(t) = \gamma \sin\left(\frac{\alpha}{\beta\hbar}\left(t - (-\beta/2)\right)\right)$$

$$\dot{c}_b(-\beta/2) = -\frac{i}{\hbar}\frac{\alpha}{\beta}\underbrace{c_a}_{\to 1} = -\frac{i}{\hbar}\frac{\alpha}{\beta}$$

$$= \frac{\alpha\gamma}{\beta\hbar}\underbrace{\cos\left(\frac{\alpha}{\beta\hbar}\left(-\beta/2-(-\beta/2)\right)\right)}_{1}$$

or $\gamma = -i$

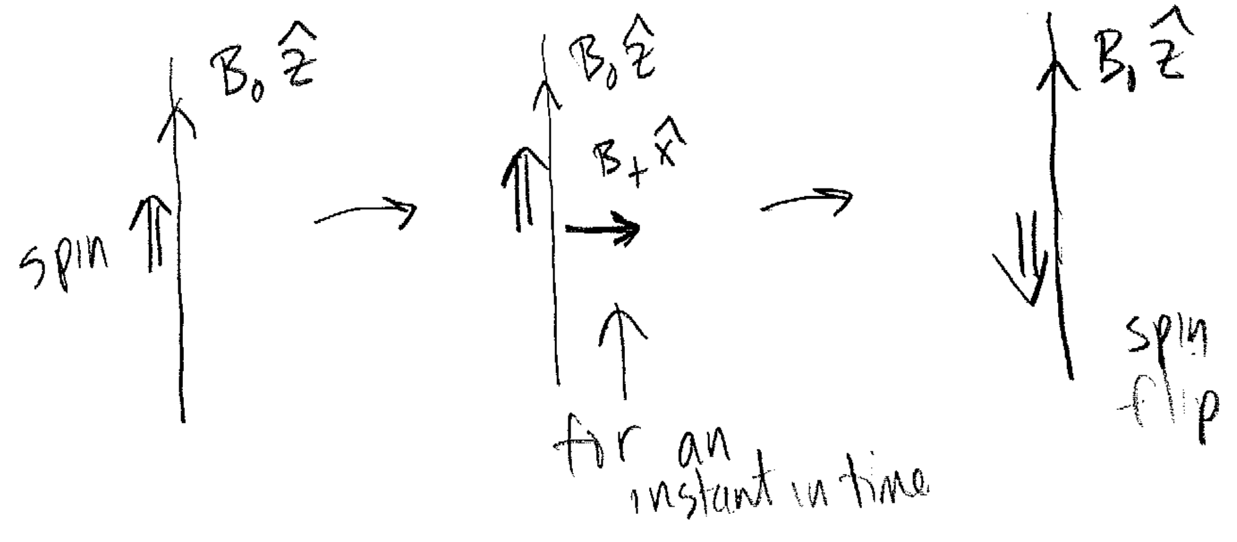
$$c_b(t) = -i\sin\left(\frac{\alpha}{\beta\hbar}\left(t-(-\beta/2)\right)\right)$$

$$c_b(\infty) = c_b(\beta/2) = -i\sin\left(\frac{\alpha}{\beta\hbar}\left(\beta/2 - -\beta/2\right)\right)$$

$$c_b(\infty) = -i\sin\left(\frac{\alpha}{\hbar}\right)$$

$$\boxed{P_{a\to b} = \sin^2\left(\frac{\alpha}{\hbar}\right)} \qquad \boxed{P_{a\to a} = 1 - P_{a\to b} = 1-\sin^2\left(\frac{\alpha}{\hbar}\right)}$$

Simple Implementation...



Master Equation(s)

2-state $\quad H'_{aa} = H'_{bb} = 0$

$$\dot{c}_a = -\frac{i}{\hbar} H'_{ab} e^{-i\omega_0 t} c_b \qquad \dot{c}_b = -\frac{i}{\hbar} H'_{ba} e^{i\omega_0 t} c_a$$

Complications in solving this . . .

MIGHT LOOK "STRAIGHTFORWARD"

since $\quad H'^{*}_{ab} = H'_{ba}$

$$\begin{pmatrix} \dot{c}_a \\ \dot{c}_b \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & -\frac{i}{\hbar} H'_{ab} e^{-i\omega_0 t} \\ -\frac{i}{\hbar} H'_{ba} e^{i\omega_0 t} & 0 \end{pmatrix}}_{\Theta(t)} \begin{pmatrix} c_a \\ c_b \end{pmatrix}$$

idea . .

$$\dot{x} = \alpha x \quad \Rightarrow \quad x = x_0 e^{\alpha t}$$

$$\dot{x} = \alpha(t) x \quad \Rightarrow \quad x = x_0 e^{\int_{t_0}^{t} \alpha(t') dt'}$$

$\boxed{\text{MAYBE}}$

$$\begin{pmatrix} c_a \\ c_b \end{pmatrix} = \begin{pmatrix} c_a(0) \\ c_b(0) \end{pmatrix} \underbrace{e^{\int_{t_0}^{t} \Theta(t') dt'}}_{\text{what's that ?}}$$

$$e^{A} = 1 + A + \frac{1}{2} A^2 + \frac{1}{6} A^3 + \cdots$$

The "exponentiation" solution OFTEN works .. but not ALWAYS. How to tell?

When $[A(t_1), A(t_2)] = 0$ for all times t_1, t_2

exponentiation <u>works</u>. Example —

when $H'_{ab}(t) = 0$, even when $H'_{aa} \neq 0$ and/or $H'_{bb}(t) = 0$. <u>WHY?</u>

$$\begin{pmatrix} \square & 0 \\ 0 & \square \end{pmatrix} \propto \underbrace{B\mathbb{I} + C\sigma_z}_{\text{always commute}}$$

when $H'_{ab}(t) \neq 0$

$$\begin{pmatrix} \square & \square \\ \square & \square \end{pmatrix} \propto B\underset{\sim}{I} + C\sigma_z + \underbrace{D\sigma_x + E\sigma_y}_{\text{off diagonals}}$$

at one time might have some non-zero σ_z, later time some non-zero σ_x ... they don't commute!

MUST USE PERTURBATION THEORY.

And, gotta be a bit careful about "Non-commutation"!

Look at $\boxed{c_a(0) = 1 \quad c_b(0) = 0}$

initially.

"Zeroth Order" leave it at that.

$\Rightarrow$ imagine $H_{aa} = H_{bb} = 0$ (often true).

Then "master equation" is

$$\dot{c}_a = \frac{-i}{\hbar} H'_{ab} e^{-i\omega_0 t} c_b$$

$$\dot{c}_b = \frac{-i}{\hbar} H'_{ba} e^{i\omega_0 t} c_a$$

Perturbation theory idea.. put lower order on right hand side to get next higher order.

First Order

$$\dot{c}_a^{(1)} = \frac{-i}{\hbar} H'_{ab} e^{-i\omega_0 t} \left(c_b^{(0)} = 0\right) = 0!$$

$$\boxed{c_a^{(1)} = 1}$$

$$\dot{c}_b^{(1)} = \frac{-i}{\hbar} H'_{ba} e^{i\omega_0 t} \left(c_a^{(0)} = 1\right) = \frac{-i}{\hbar} H'_{ba} e^{i\omega_0 t}$$

So, $\boxed{c_b^{(1)}(t) = 0 - \frac{i}{\hbar} \int_0^t H'_{ba}(t') e^{i\omega_0 t} dt'}$

"Turn the crank"

$$\dot{c}_a^{(2)} = -\frac{i}{\hbar} H'_{ab} e^{-i\omega_0 t} c_b^{(1)}$$

or
$$\frac{dc_a^{(2)}}{dt} = \underline{-\frac{i}{\hbar} H'_{ab} e^{-i\omega_0 t}} \left(-\frac{i}{\hbar} \int_0^t H'_{ba}(t') e^{i\omega_0 t'} dt' \right)$$

stare at this!

$\boxed{\text{one instant in time}}$

integral over EARLIER TIMES

$$c_a^{(2)} = \underset{\text{initial condition}}{1} + \underbrace{\left(-\frac{i}{\hbar} \int_0^t H'_{ab}(t') e^{-i\omega_0 t'} \left(-\frac{i}{\hbar} \int_0^{t'} H'_{ba}(t'') e^{i\omega_0 t''} \right) dt' \right)}$$

<u>Not</u> the same as...

$$-\frac{1}{\hbar^2} \int_0^t \int_0^t dt' dt'' \underline{\quad \text{stuff} \quad}$$

THIS t' REALLY MATTERS.