Chapter 9: $\underline{\underline{\text{Statics}}} \longrightarrow$ Dynamics.

Statics $\quad H\psi = i\hbar \frac{\partial \psi}{\partial t}$

$$\frac{p^2}{2m} + \underline{\underline{V(\vec{r})}} \leftarrow \text{static.}$$

Solution: $\quad \psi(\vec{r},t) = \psi(\vec{r})e^{-iEt/\hbar}$

$$|\psi(\vec{r},t)|^2 = \underbrace{|\psi(\vec{r})|^2}_{\text{time independent.}}$$

How to start describing time dependence?

(1) Initial state not a pure eigen state... eg, $\underline{\text{2-state system}}$

$$H^0|\psi_a\rangle = E_a|\psi_a\rangle \quad H^0|\psi_b\rangle = E_b|\psi_a\rangle$$

$$(E_a \neq E_b)$$

$t=0 \quad |\psi(0)\rangle = c_a|\psi_a\rangle + c_b|\psi_b\rangle$] c_a, c_b real

$$c_a^2 + c_b^2 = 1$$

$t \neq 0 \quad |\psi(t)\rangle = c_a e^{\frac{-iE_a t}{\hbar}}|\psi_a\rangle + c_b e^{\frac{-iE_b t}{\hbar}}|\psi_b\rangle$

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actually, still <u>little</u> time dependence

$\langle \Psi(t) | \Psi(t) \rangle \rightarrow$ key point is

$$\langle \Psi_a | \Psi_b \rangle = \delta_{ab}$$

$$= c_a^2 + c_b^2 = 1$$

$$\langle \Psi_a | \Psi(t) \rangle = \underbrace{c_a e}^{-iE_a t/\hbar}$$

<u>magnitude</u> time independent.

<u>phases</u> hard to measure.

<u>Comment</u>: NOT in TEXT

<u>Sometimes</u> measurement apparatus responds to ...

$$\alpha | \Psi_a \rangle + \beta | \Psi_b \rangle$$

Most famous example
"neutrino mixing"

charged leptons

detectors respond to $\nu_e \Leftrightarrow e^-$

$\nu_\mu \Leftrightarrow \mu^-$

Eigenstates of H are not ν_e, ν_μ, but called ν_1, ν_2

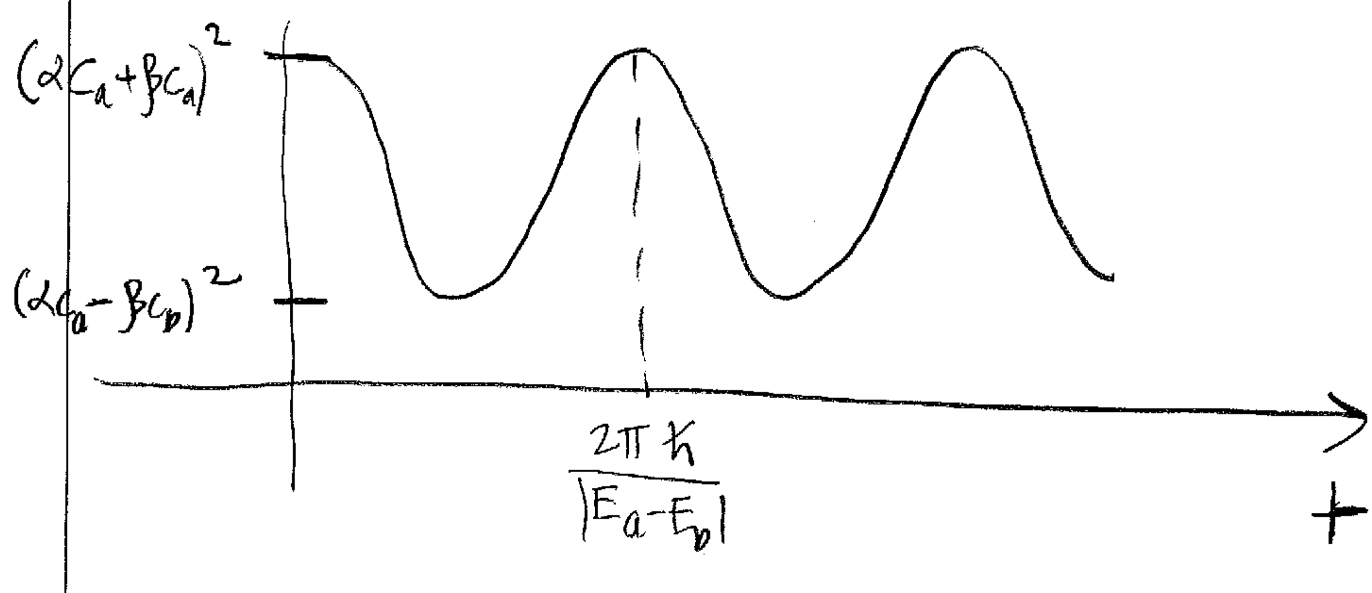
$$\nu_e = \alpha|\psi_a\rangle + \beta|\psi_b\rangle \qquad \alpha, \beta \text{ real}$$

$$\langle \nu_e | \Psi(t)\rangle = \left[\alpha\langle\psi_a| + \beta\langle\psi_b|\right]\left[c_a e^{\frac{-iE_a t}{\hbar}}|\psi_a\rangle + c_b e^{\frac{-iE_b t}{\hbar}}|\psi_b\rangle\right]$$

$$= \alpha c_a e^{\frac{-iE_a}{\hbar}t} + \beta c_b e^{-iE_b t/\hbar}$$

$$|\langle \nu_e|\Psi(t)\rangle|^2$$

$$= \alpha^2 c_a^2 + \beta^2 c_b^2 + 2\alpha\beta c_a c_b \underbrace{\mathrm{Re}\left[e^{i(E_a - E_b)t/\hbar}\right]}_{\text{bounces between } +1 \text{ \& } -1}$$



Back to Book: Introduce time-d perturb.

$$H|\psi\rangle = i\hbar \frac{\partial}{\partial t}|\psi\rangle$$

$$\underbrace{H = H^0 + H'(t)}$$

best to visulize in 2d representation

$$H^0 + H'(t) \rightarrow \begin{bmatrix} E_a & 0 \\ 0 & E_b \end{bmatrix} + \begin{bmatrix} H'_{aa} & H'_{ab} \\ H'_{ba} & H'_{bb} \end{bmatrix}$$

$$H'_{ba} = H'^{*}_{ab}$$

$$c_a e^{\frac{-iE_a t}{\hbar}}|\psi_a\rangle + c_b e^{\frac{-iE_b t}{\hbar}}|\psi_b\rangle \rightarrow \begin{bmatrix} c_a e^{-iE_a/\hbar t} \\ c_b e^{-iE_b/\hbar t} \end{bmatrix}$$

$$H|\psi\rangle \rightarrow \underbrace{\left\{ \begin{bmatrix} E_a & 0 \\ 0 & E_b \end{bmatrix} + \begin{bmatrix} H'_{aa} & H'_{ab} \\ H'_{ba} & H'_{bb} \end{bmatrix} \right\}}_{H^0 + H'(t)} \begin{bmatrix} c_a e^{\frac{-iE_a}{\hbar}t} \\ c_b e^{\frac{-iE_b}{\hbar}t} \end{bmatrix}$$

LHS

$$\left\{ \begin{bmatrix} E_a c_a e^{\frac{-iE_a}{\hbar}t} \\ E_b c_b e^{\frac{-iE_b}{\hbar}t} \end{bmatrix} + \underbrace{\begin{bmatrix} H'_{aa} & H'_{ab} \\ H'_{ba} & H'_{bb} \end{bmatrix}}_{\text{might depend on time!}} \begin{bmatrix} c_a e^{\frac{-iE_a}{\hbar}t} \\ c_b e^{\frac{-iE_b}{\hbar}t} \end{bmatrix} \right\}$$

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RHS

$$i\hbar\frac{\partial}{\partial t}|\psi\rangle \rightarrow i\hbar\frac{\partial}{\partial t}\begin{bmatrix} c_a e^{-iE_a/\hbar\, t} \\ c_b e^{-iE_b/\hbar\, t} \end{bmatrix}$$

c_a + c_b might be time dependent!

$$= \begin{bmatrix} i\hbar \dot{c}_a e^{-\frac{iE_a}{\hbar}t} + E_a c_a e^{-\frac{iE_a}{\hbar}t} \\ i\hbar \dot{c}_b e^{-\frac{iE_b}{\hbar}t} + \underbrace{E_b c_b e^{-\frac{iE_b}{\hbar}t}}_{\text{CANCELS!}} \end{bmatrix}$$

NOW BE CAREFUL

$$\begin{bmatrix} H'_{aa} & H'_{ab} \\ H'_{ba} & H'_{bb} \end{bmatrix}\begin{bmatrix} c_a e^{-\frac{iE_a}{\hbar}t} \\ c_b e^{-\frac{iE_b}{\hbar}t} \end{bmatrix} = i\hbar\begin{bmatrix} \dot{c}_a e^{-\frac{iE_a}{\hbar}t} \\ \dot{c}_b e^{-\frac{iE_b}{\hbar}t} \end{bmatrix}$$

think of this as driven by