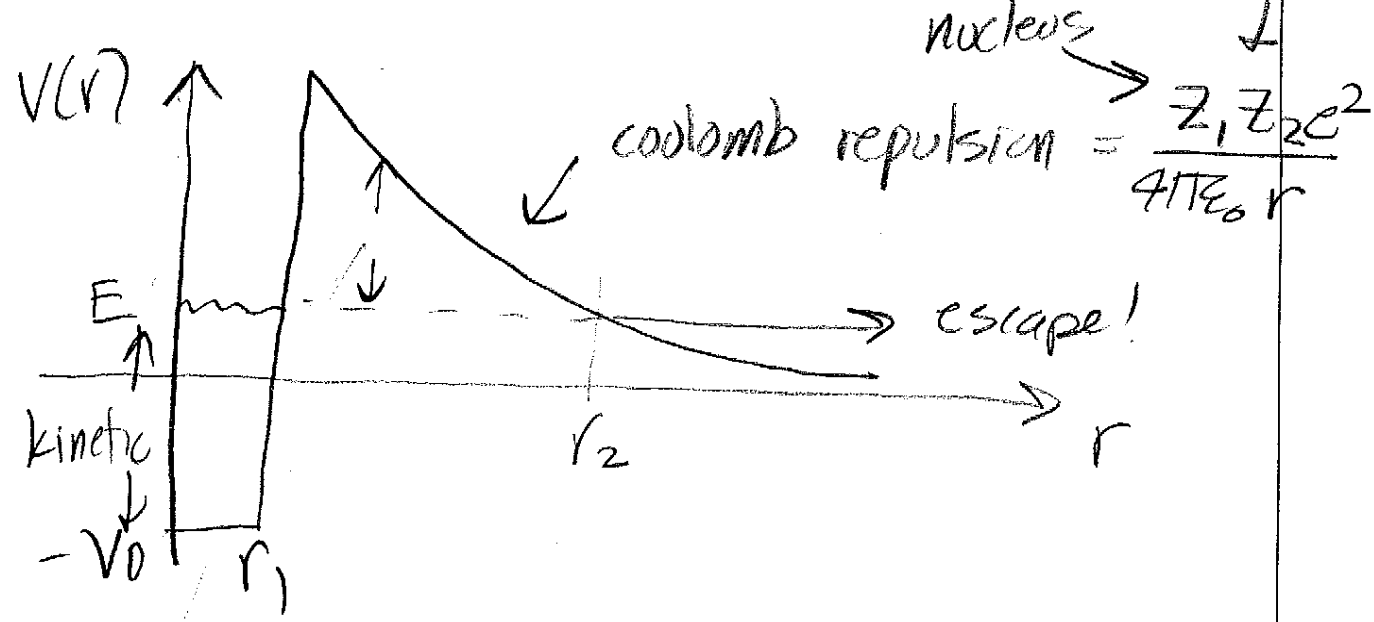
Specific α-decay



TURNING POINTS:

$r_1 \Rightarrow$ constant, unknown from nuclear physics

$r_2 \Rightarrow$ determined by (E → measured!)

$$\frac{1}{4\pi\varepsilon_0}\frac{Z_1 Z_2 e^2}{r_2} = E$$

$$r_2 = \frac{1}{4\pi\varepsilon_0}\frac{Z_1 Z_2 e^2}{E}$$

$$\left|\frac{p^2}{2m}\right| = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0 r} - E$$

$$|p| = \sqrt{2m\left(\frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0 r} - E\right)}$$

$$T \simeq e^{-2\gamma}$$

$$\gamma = \frac{1}{\hbar}\int_{r_1}^{r_2}\sqrt{2m\left(\frac{1}{4\pi\varepsilon_0}\frac{Z_1 Z_2 e^2}{r} - E\right)}\,dr$$

$$= \frac{\sqrt{2mE}}{\hbar}\int_{r_1}^{r_2}\sqrt{\frac{1}{4\pi\varepsilon_0}\frac{Z_1 Z_2 e^2}{E r} - 1}\,dr$$

$$= \frac{\sqrt{2mE}}{\hbar}\int_{r_1}^{r_2}\sqrt{\frac{r_2}{r} - 1}\,dr$$

$$r = r_2 \sin^2 u \quad \Bigg] \quad \frac{r = r_2,\ u = \pi/2}{r = r_1,\ u = \sin^{-1}\left(\sqrt{\frac{r_1}{r_2}}\right)}$$

$$dr = 2r_2 \sin u \cos u$$

$$= \frac{\sqrt{2mE}}{\hbar}\int_{\sin^{-1}\left(\sqrt{\frac{r_1}{r_2}}\right)}^{\pi/2}\sqrt{\frac{1}{\sin^2 u} - 1}\ 2r_2 \sin u \cos u\,du$$

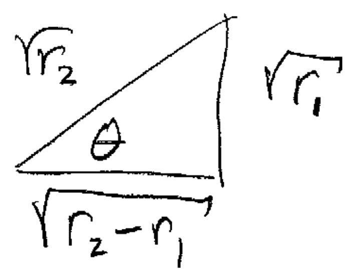
$$= \frac{\sqrt{2mE}}{\hbar}\,2r_2\int_{\sin^{-1}\left(\sqrt{\frac{r_1}{r_2}}\right)}^{\pi/2}\sqrt{1 - \sin^2 u}\ \cos u\,du$$

$$= 2r_2\frac{\sqrt{2mE}}{\hbar}\int_{\sin^{-1}\left(\sqrt{r_1/r_2}\right)}^{\pi/2}\cos^2 u\,du$$

$$= 2r_2 \frac{\sqrt{2mE}}{\hbar} \left(\frac{u}{2} + \frac{1}{4}\sin(2u) \right]_{\sin^{-1}(\sqrt{r_1/r_2})}^{\pi/2} \Bigg)$$

$$= 2r_2 \frac{\sqrt{2mE}}{\hbar} \left(\frac{\pi}{4} - \frac{1}{2}\sin^{-1}\sqrt{\frac{r_1}{r_2}} + \frac{1}{4}\underbrace{\cancel{\sin(\pi)}}_{0} - \frac{1}{4}\sin\left(2\sin^{-1}\left(\sqrt{\frac{r_1}{r_2}}\right)\right) \right)$$

$$\sin\left(2\sin^{-1}\left(\sqrt{\frac{r_1}{r_2}}\right)\right) = 2\sin\left(\sin^{-1}\left(\sqrt{\frac{r_1}{r_2}}\right)\right)\cos\left(\sin^{-1}\left(\sqrt{\frac{r_1}{r_2}}\right)\right)$$



$$\sin\theta = \sqrt{\frac{r_1}{r_2}}$$

$$\cos\theta = \sqrt{1 - \frac{r_1}{r_2}}$$

$$= 2\sqrt{\frac{r_1}{r_2}\left(1 - \frac{r_1}{r_2}\right)}$$

$$= 2r_2 \frac{\sqrt{2mE}}{\hbar} \left(\frac{\pi}{4} - \frac{1}{2}\sin^{-1}\sqrt{\frac{r_1}{r_2}} - \frac{1}{4}\cdot 2\sqrt{\frac{r_1}{r_2}\left(1 - \frac{r_1}{r_2}\right)} \right)$$

$$= \frac{\sqrt{2mE}}{\hbar} \left(r_2\left(\frac{\pi}{2} - \sin^{-1}\sqrt{\frac{r_1}{r_2}} \right) - \sqrt{r_1(r_2 - r_1)} \right)$$

$$r_1 \ll r_2 \rightarrow \sin^{-1}\sqrt{\frac{r_1}{r_2}} = \sqrt{\frac{r_1}{r_2}}$$

$$\sqrt{r_1(r_2 - r_1)} = \sqrt{r_1 r_2}$$

$$\gamma \simeq \frac{\sqrt{2mE}}{\hbar}\left(\frac{\pi}{2} r_2 - 2\sqrt{r_1 r_2} \right) \simeq \underbrace{\frac{\sqrt{2mE}}{\hbar} \cdot \frac{\pi}{2} \cdot \frac{Z_1 Z_2 e^2}{E\, 4\pi\epsilon_0}}_{\text{simpler than Griffiths}}$$

<u>set $r_1 \rightarrow 0$!</u>

$$\sqrt{\frac{2mE}{E^2}} = \sqrt{\frac{2m}{E}} = \sqrt{\frac{2\cancel{m}}{\frac{1}{2}\cancel{m}v^2}} = \frac{2}{v}$$

$$\gamma \cong \frac{\cancel{2}}{v} \cdot \frac{\pi}{\cancel{2}} \cdot \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0 \hbar}$$

$$\simeq \pi \cdot \underbrace{\frac{c}{v}}_{\frac{1}{\beta}} \cdot Z_1 Z_2 \frac{e^2}{4\pi\varepsilon_0 \hbar c}$$

Fine Structure constant, α

of α (relativity)

Gamow Factor

$$\text{"}G\text{"} = \gamma = \pi \cdot \frac{\alpha Z_1 Z_2}{\beta}$$ Dimensionless

β: E from 4–8 MeV

α particle: $m_\alpha c^2 \simeq 3727$ MeV

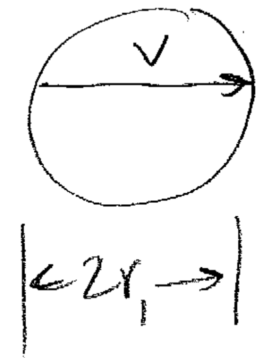
$$E = \frac{1}{2} m_\alpha v^2 = \frac{1}{2} m_\alpha c^2 \cdot \beta^2$$

$$\sqrt{\frac{4}{\frac{1}{2} 3727}} < \beta < \sqrt{\frac{8}{\frac{1}{2} \cdot 3727}}$$

$$0.046 < \beta < .065$$

~ Non-Relativistic

Convert to lifetime:



$$v\Delta t = 2r_1$$

$\underline{\text{frequency try}}$ $\quad \dfrac{1}{\Delta t} = \dfrac{2r_1}{v}$

$$\text{frequency success} = \frac{2r_1}{v}\cdot e^{-2\gamma}$$

$$\left.\begin{matrix}\text{Mean}\\ \text{Time}\\ \text{Between}\\ \text{Successes}\end{matrix}\right\} = \frac{v}{2r_1}\, e^{2\gamma}$$

$$= \frac{v}{2r_1}\cdot e^{-2G}$$

$$= \frac{v}{2r_1}\cdot \underbrace{e^{-2\pi\cdot\frac{\alpha Z_1 Z_2}{\beta}}}_{\text{dominant factor.}}$$

$Z_1 \cong 90$ $\quad \underline{\beta = 0.046}$ $\quad \underline{\beta = 0.065}$

$Z_2 \simeq 2$

$2G$ $\quad \underbrace{179.5 \qquad 127.0}$

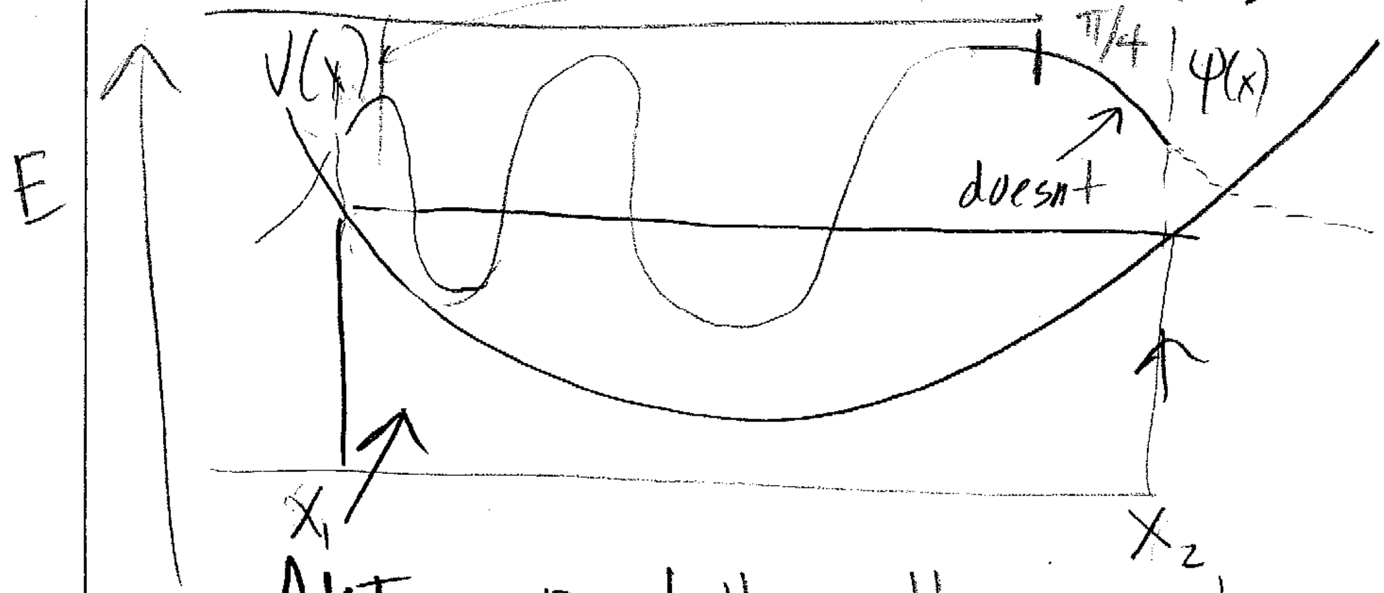
$$\Delta:\; e^{+(179.5-127.0)} = e^{+52.5} = (10^{0.43})^{52.5} = 10^{+23}!$$

FACTOR OF $\simeq 1.5$ in β

$\simeq$ 20 orders of magnitude in mean life!

"GAMOW FACTOR"

Connection Formulas WKB



NOT ∞ tall wall, smooth

$$\frac{1}{\hbar}\int_{r_1}^{r_2} p\,dx \neq n\pi, \text{ but, less!}$$

Two smooth walls:

$$\frac{1}{\hbar}\int_{x_1}^{x_2} p(x)\,dx = \left(n - \frac{1}{4} - \frac{1}{4}\right)\pi$$
$$= \left(n - \frac{1}{2}\right)\pi$$

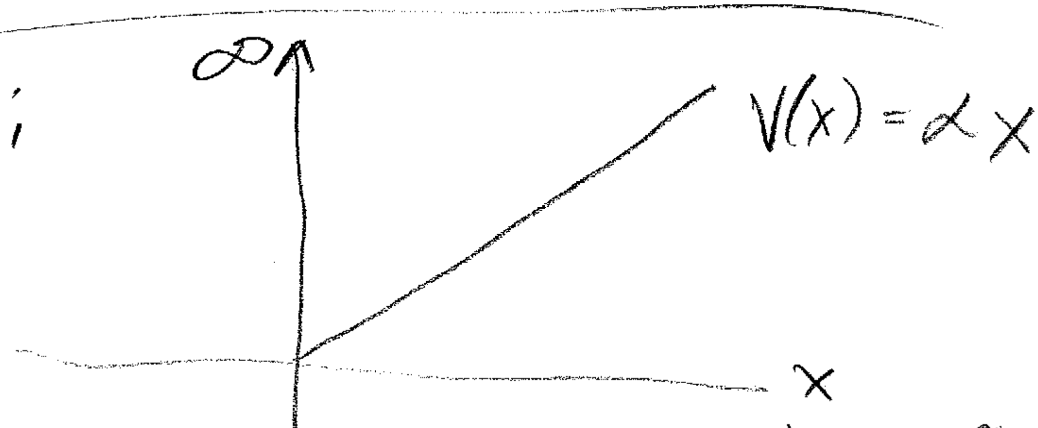
One ∞ wall, one smooth wall.

$$\frac{1}{\hbar}\int_{x_1}^{x_2} p(x)\,dx = \left(n - \frac{1}{4}\right)\pi$$

Two ∞ walls

$$\frac{1}{\hbar}\int_{x_1}^{x_2} p(x)\,dx = n\pi$$

8.6;



$$p(x) = \sqrt{2m(E_n - \alpha x)}$$

turning points:
$x_1 = 0$
$x_2 = \frac{E_n}{\alpha}$

$$\frac{1}{\hbar}\int_0^{E_n/\alpha}\sqrt{2m(E_n-\alpha x)}\,dx = \left(n-\frac{1}{4}\right)\pi$$

$$\frac{d}{dx}\left[\frac{2}{3}(E_n-\alpha x)^{3/2}\left(-\frac{1}{\alpha}\right)\right] = \cancel{\frac{2}{3}}\cancel{\frac{3}{2}}(E_n-\alpha x)^{1/2}\left(\frac{-\alpha}{-\alpha}\right)$$

so $$-\frac{\sqrt{2m}}{\hbar}\frac{2}{3}(E_n-\alpha x)^{3/2}\Big|_0^{E_n/\alpha} = \left(n-\frac{1}{4}\right)\pi$$

$$-\frac{\sqrt{2m}}{\hbar}\cdot\frac{2}{3}\left[0-E_n^{3/2}\right] = \left(n-\frac{1}{4}\right)\pi$$

$$\boxed{E_n = \left[\frac{3}{2}\frac{\hbar}{\sqrt{2m}}\left(n-\frac{1}{4}\right)\pi\right]^{2/3}}$$