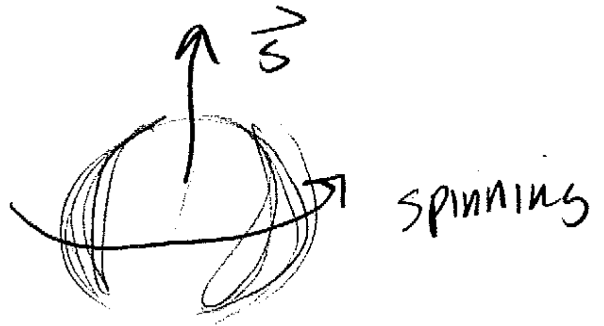
Hyperfine

What about the spin of the nucleus?

Hydrogen → proton



$r_p \simeq 10^{-13}$ cm

$\sim 10^{-5} \times a_0$

$(m_p \sim 2000 \cdot m_e)$

Hmm... maybe

$\mu_p = \frac{+e}{2m_p} \cdot \left(\frac{\hbar}{2}\right)$

NOPE!

$= \frac{g_p e}{2m_p} \left(\frac{\hbar}{2}\right)$

$\underline{g_p = 5.59}$!

Electron → "point particle"

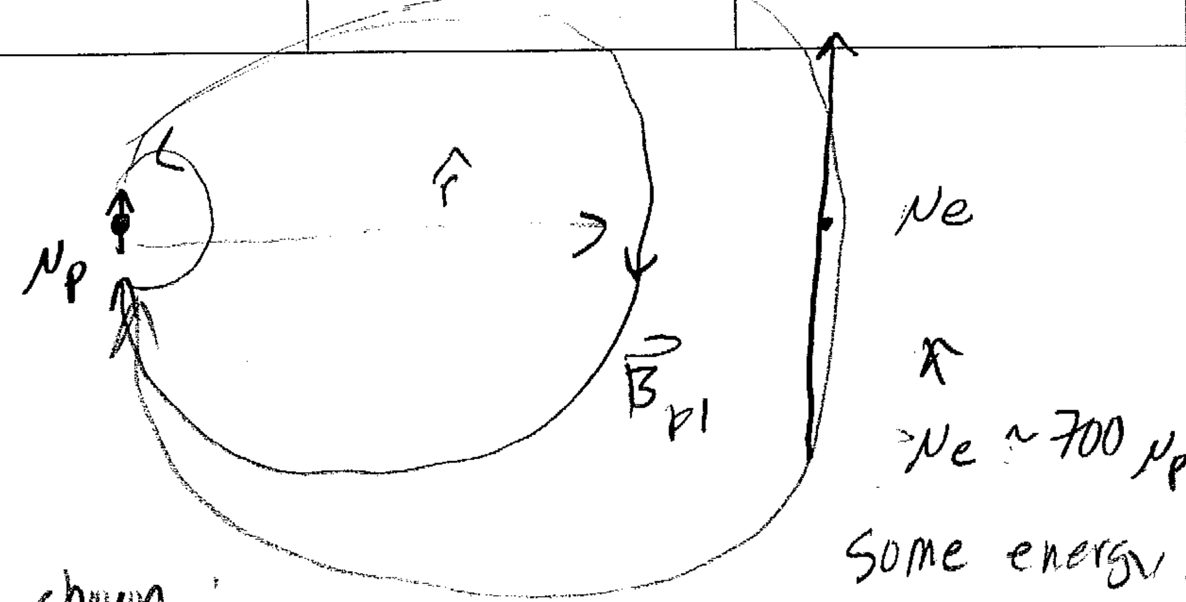
$g_e \simeq 2$ (!)

Proton → "composite" (3-quarks)

$g_p \simeq 5.6$ (!)

Neutron? $g_n \simeq -3.83$ (!)

Electrically neutral!



as shown :

$$\vec{B}_{p1} = \frac{\mu_0}{4\pi}\left[3(\vec{\mu}_p \cdot \hat{r})\hat{r} - \vec{\mu}_p\right]$$

when $\hat{r}\cdot\vec{\mu}_p = 0$

$\vec{B}_{p1} \uparrow\downarrow \vec{\mu}$

(as shown).

Start easy

$$\langle \Psi_{n00} | \vec{\mu}_e \cdot \vec{B}_{p1} | \Psi_{n00} \rangle = 0!$$

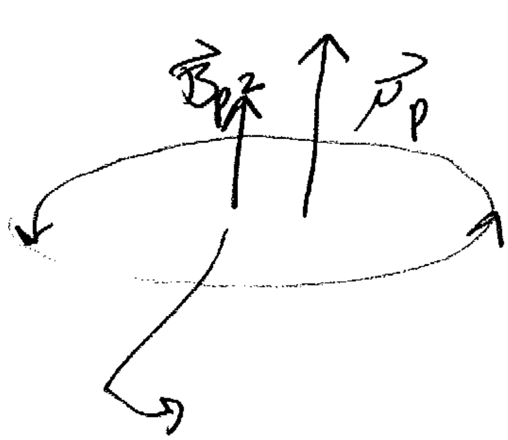
all same

why? $|\Psi_{n00}|^2$ is spherically symmetric.

$\vec{\mu}_{p1} \cdot \vec{B}_{p1}$ is... a "rank 2" object (!)

However, here's a fun thing . . .

open up the proton. (loop of current



$$\vec{B}_{pz} = \frac{2\mu_0}{3}\vec{\mu}_p \underbrace{\delta^3(\vec{r})}_{\text{must sit on proton.}}$$

$$-\vec{\mu}_e \cdot \vec{B}_{pz}$$

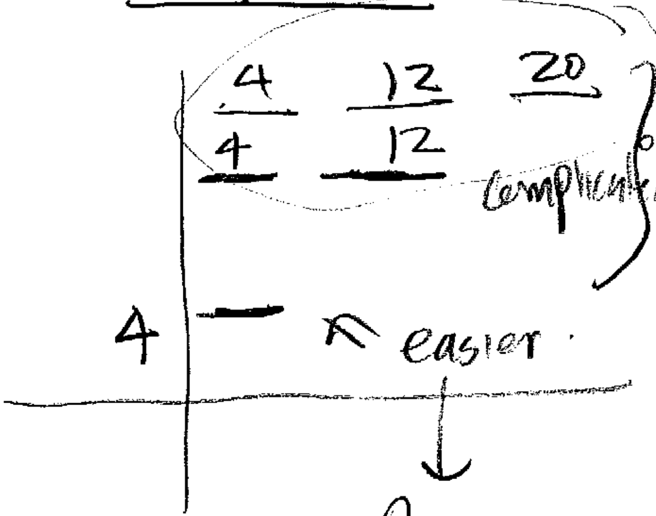
$$= \underbrace{\frac{\mu_0 g_p e^2}{3 m_p m_e} \vec{S}_p \cdot \vec{S}_e \, \delta^3(\vec{r})}$$

only non-zero E^1_{hf} when $l=0$ (sitting on origin)

$$\langle \psi_{n00} | -\vec{\mu}_e \cdot \vec{B}_{pz} | \psi_{n00} \rangle$$

$$= \frac{\mu_0 g_p e^2}{3 m_p m_e} \langle \vec{S}_p \cdot \vec{S}_e \rangle \underbrace{|\psi_{n00}|^2}_{\frac{1}{n^3 \pi a^3}}$$

$\langle \vec{S}_p \cdot \vec{S}_e \rangle$ Include Proton Spin Now



even more complicated

easier

Rearranges into eigenstates of $\vec{S} = \vec{S}_e + \vec{S}_p$

$$\vec{S} = \vec{S}_e + \vec{S}_p$$

$$\uparrow \qquad \uparrow \qquad \uparrow$$

$$1 \text{ or } 0 \qquad \tfrac{1}{2} \qquad \tfrac{1}{2}$$

$$3 \qquad 1$$

Triplet Singlet.

$$\vec{S}_p \cdot \vec{S}_e = \frac{1}{2}\left(S^2 - S_e^2 - S_p^2\right) \quad \text{Again!}$$

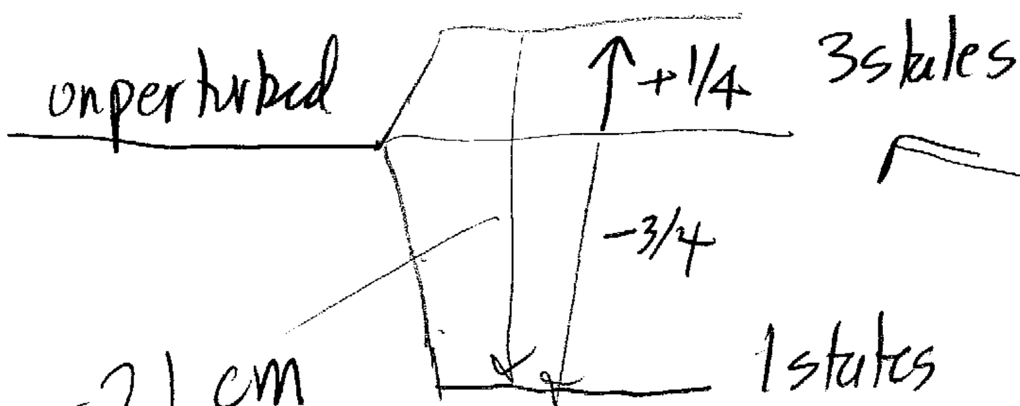
$$= \frac{1}{2}\left(S(S+1) - \frac{3}{2}\right)\hbar^2$$

$$= -\frac{3}{4}\hbar^2 \qquad S=0$$

$$+\frac{1}{4}\hbar^2 \qquad S=1$$

$$E^1_{hf} = \frac{4 g_p \hbar^4}{3 m_p m_e^2 c^2 a^4} \begin{cases} +1/4 & \text{triplet} \\ -3/4 & \text{singlet} \end{cases}$$

$$\Delta E = 5.8 \cdot 10^{-6}\ eV$$



$\lambda = 21$ cm

Very Penetrating

Mapped The Galaxy

Barycenter Unperturbed.

when

$$Tr(\langle \psi_n | H' | \psi_m \rangle) = 0.$$

Chapter 7: The Variational Principle.

Point is: • guess $\psi_g(x)$, know H

$$E_{gs} \leq \langle \psi_g | H | \psi_g \rangle \qquad \langle \psi_g | \psi_g \rangle = 1$$

GROUND STATE from your guess!

Proof:
$$|\psi_g\rangle = \sum_n c_n |\psi_n\rangle$$

true, but unknown eigenfunctions of H

$$\langle \psi_g | \psi_g \rangle = 1 = \left(\sum_m c_m^* \langle \psi_m | \right) \left(\sum_n c_n |\psi_n\rangle \right)$$

$$= \sum_{m,n} c_m^* c_n \underbrace{\langle \psi_m | \psi_n \rangle}_{\delta_{mn}}$$

$$1 = \sum_n |c_n|^2$$

$$\langle \psi_g | H | \psi_g \rangle = \left(\sum_m c_m^* \langle \psi_m | \right) H \left(\sum_n c_n |\psi_n\rangle \right)$$

$$= \sum c_m^* c_n \begin{pmatrix} \text{either } E_m \\ \text{or } E_n \end{pmatrix} \underbrace{\langle \psi_m | \psi_n \rangle}_{\delta_{mn}}$$

$$= \sum_n |c_n|^2 E_n \qquad E_n \geq E_{gs}:$$

$$\therefore \langle \psi_g | H | \psi_g \rangle \geq E_{gs} \underbrace{\sum_n |c_n|^2}_{1}$$

$$\langle \psi_g | H | \psi_g \rangle \geq E_{gs}$$

<u>Example</u> 1-d

$$H = \frac{-\hbar^2}{2m} \frac{d}{dx^2} + \alpha |x|^n$$

$$= T + V \qquad \frac{1}{2} m\omega^2 \text{ for SHO}$$

<u>guess</u>:

$$\psi_g(x) = A e^{-bx^2} \quad \underline{\text{guess}} \quad \left\{ \begin{array}{l} b \\ \text{adjustable} \end{array} \right.$$

$$\boxed{\text{CHECK NORMALIZATION}}$$

$$1 = |A|^2 \int_{-\infty}^{\infty} e^{-2bx^2} dx = |A|^2 \sqrt{\frac{\pi}{2b}} \quad (\text{given})$$

$$A = \left(\frac{2b}{\pi}\right)^{1/4}$$

$$\langle \psi_g | H | \psi_g \rangle = \langle T \rangle + \langle V \rangle$$

$$\langle T \rangle = -\frac{\hbar^2}{2m} \left(\frac{2b}{\pi}\right)^{1/2} \int_{-\infty}^{\infty} e^{-bx^2} \frac{d^2}{dx^2} e^{-bx^2} dx$$

$$\frac{d}{dx} e^{-bx^2} = e^{-bx^2} \frac{d}{dx}(-bx^2)$$

$$= -2bx\, e^{-bx^2}$$

$$\frac{d^2}{dx^2} e^{-bx^2} = -2b\, e^{-bx^2} + 4b^2x^2 e^{-bx^2}$$

$$\int_{-\infty}^{\infty} e^{-2bx^2}(-2b)\,dx = -2b\left(\frac{\pi}{2b}\right)^{1/2}$$

$$= -2\sqrt{\frac{\pi}{2}}\; b^{1/2}$$

$$\int_{-\infty}^{\infty} e^{-2bx^2} 4b^2x^2\,dx$$

hmm.. if $\int_{-\infty}^{\infty} e^{-2bx^2}\,dx = \sqrt{\frac{\pi}{2b}}$

$$\frac{d}{db}\int_{-\infty}^{\infty} e^{-2bx^2}\,dx = -2\int_{-\infty}^{\infty} x^2 e^{-2bx^2}\,dx$$

$$= \frac{d}{db}\left[\sqrt{\frac{\pi}{2}}\; b^{-1/2}\right]$$

$$= -\frac{1}{2}\sqrt{\frac{\pi}{2}}\; b^{-3/2}$$

so $\int_{-\infty}^{\infty} x^2 e^{-2bx^2}\,dx = \frac{1}{4}\sqrt{\frac{\pi}{2}}\;\frac{1}{b^{3/2}}$