Hydrogen Binding Energies... $O(\alpha^2 mc^2)$

Fine Structure... Relativistic... $O(\alpha^4 mc^2)$

"Breaks degeneracies"

"$j = l \pm \frac{1}{2}$ good quantum #"

Lamb Shift... $O(\alpha^5 mc^2)$

"Breaks j degeneracy, depending on which l"

All these are <u>much larger</u> in bound states of nucleons, quarks

Zeeman effect... put atom in an <u>external</u> B_{ext} field

Stark effect.. put atom in an <u>external</u> E_{ext} field. (homework)

$\vec{L}\cdot\vec{S}$ ... atom has its <u>own</u> $\vec{B}_{int}$ field.. key question...

is $\vec{B}_{ext} \gtrless \vec{B}_{int}$?

$>$.. strong field] eigenfunctions of $L_z + 2S_z$ (L^2, S^2)

$<$ weak field] J^2, J_z

Zeeman :

$$H'_z = -(\vec{\mu}_L + \vec{\mu}_S)\cdot \vec{B}_{ext}$$

$\vec{\mu}_L \to \underbrace{-\frac{e}{2m}\vec{L}}_{\text{"usual"}}$ $\quad$ $\vec{\mu}_S \to \underbrace{-g_e\frac{e}{2m}\vec{S}}_{\text{anamolous}}$ $\quad$ $\vec{B}_{ext}$: usually choose z direction

$g_e = 2.0023193043615$
$\pm .0000000000006$

$$H'_z \simeq +\frac{e}{2m}\underbrace{(\vec{L}+2\vec{S})}_{\text{not } \vec{J}=\vec{L}+\vec{S}}\cdot \vec{B}_{ext}$$

Cases: Easier ... strong field

$$H'_z \simeq \frac{e}{2m}\underbrace{(L_z + 2S_z)}_{\text{this matters}} B_{ext}$$

use eigenfunctions of

$L^2, L_z ; S^2, S_z$

Zeeman

$$H_Z' = -(\vec{\mu}_L + \vec{\mu}_s) \cdot \vec{B}_{ext}$$

$$\vec{\mu}_L = -\frac{e}{2m}\vec{L} \qquad \vec{\mu}_s = -g_e \frac{e}{2m}\vec{S}$$

$g_e = 2.00231930043615$
$\pm 0.0 \ldots 06$

Key point: how does Zeeman energy compare with fine structure energy?

$$\frac{e\hbar}{2m} \simeq 5.8 \cdot 10^{-5} \frac{eV}{T}$$

Compare: $$\frac{e\hbar}{2m} B_{ext} = \alpha^2 \frac{E_1}{n^2}$$

$$B_{ext} = \frac{\left(\frac{1}{137}\right)^2 \cdot \frac{13.6}{n^2}}{5.8 \cdot 10^{-5} \frac{eV}{T}}$$

$$B_{ext} = \frac{12.5\ T}{n^2}$$

~ 25 T available at National High Field Lab.

Strong Field : • easier

• treat H_{fs} as a perturbation ..

$$H_Z' = \frac{e}{2m}(L_z + \underset{\substack{\uparrow \\ \text{approx}}}{2} S_z) B_{ext}$$

"Good" eigenstates?

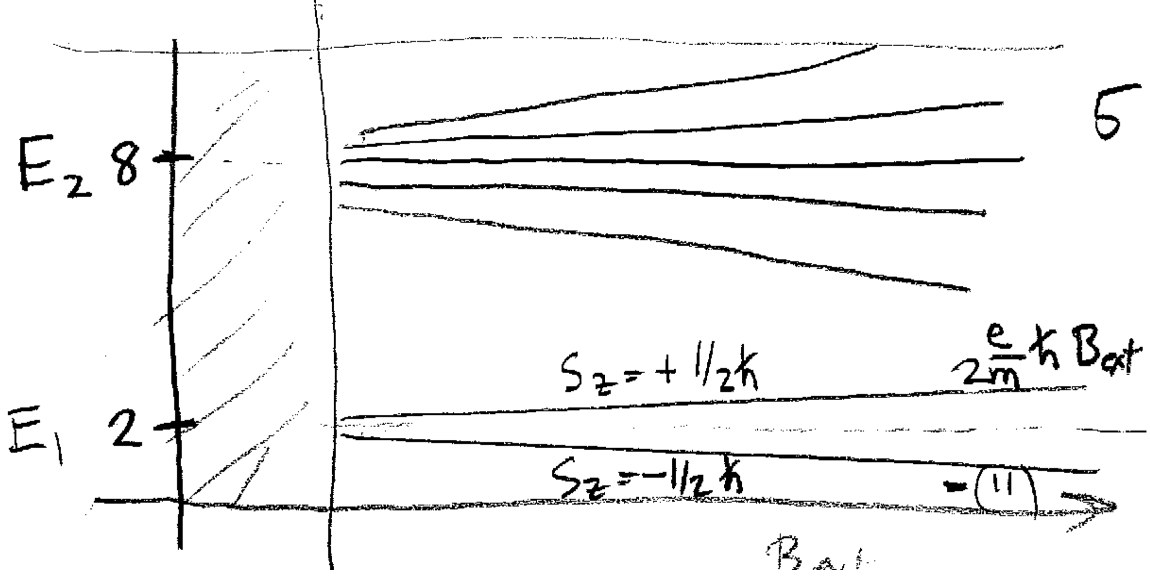
"Fine Structure" ⇒ J^2, L^2, S^2, J_z "total J"] $j = l \pm \frac{1}{2}$

"Strong Field" ⇒ L^2, S^2, L_z, S_z "product states"

$\boxed{\text{key point}}$ → $[J^2, L_z] \neq 0$

$[J^2, S_z] \neq 0$

In the limit that B_{ext} dominates ...



$$8: \quad \left.\begin{array}{l} L_z = 1, 0, -1 \\ 2S_z = 1, -1 \end{array}\right\} \quad \underline{\text{combine}}: \quad \begin{array}{ll} 2 & \text{(1 way)} \\ 1 & \text{(2 ways)} \\ 0 & \text{(2 ways)} \\ -1 & \text{(2 ways)} \\ -2 & \text{(1 way)} \end{array}$$

→ magnetic field <u>splits into 5</u>

→ what about the degeneracies at $L + 2_z = \pm 1, 0$?

→ maybe split by — fine structure!

now, $\vec{S} \cdot \vec{L}$ <u>not</u> <u>strong</u> enough to force its eigenstates...

want

$$\langle n l m_l m_s | \underbrace{\vec{S} \cdot \vec{L}}_{} | n l m_l m_s \rangle$$

means $S_x L_x + S_y L_y + S_z L_z$

$$\langle m_s | S_x | m_s \rangle = \langle m_s | S_y | m_s \rangle = 0$$

recall $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

<u>But</u> $\quad \langle \, | S_z L_z | \, \rangle = \hbar^2 m_l m_s$

who gets split?

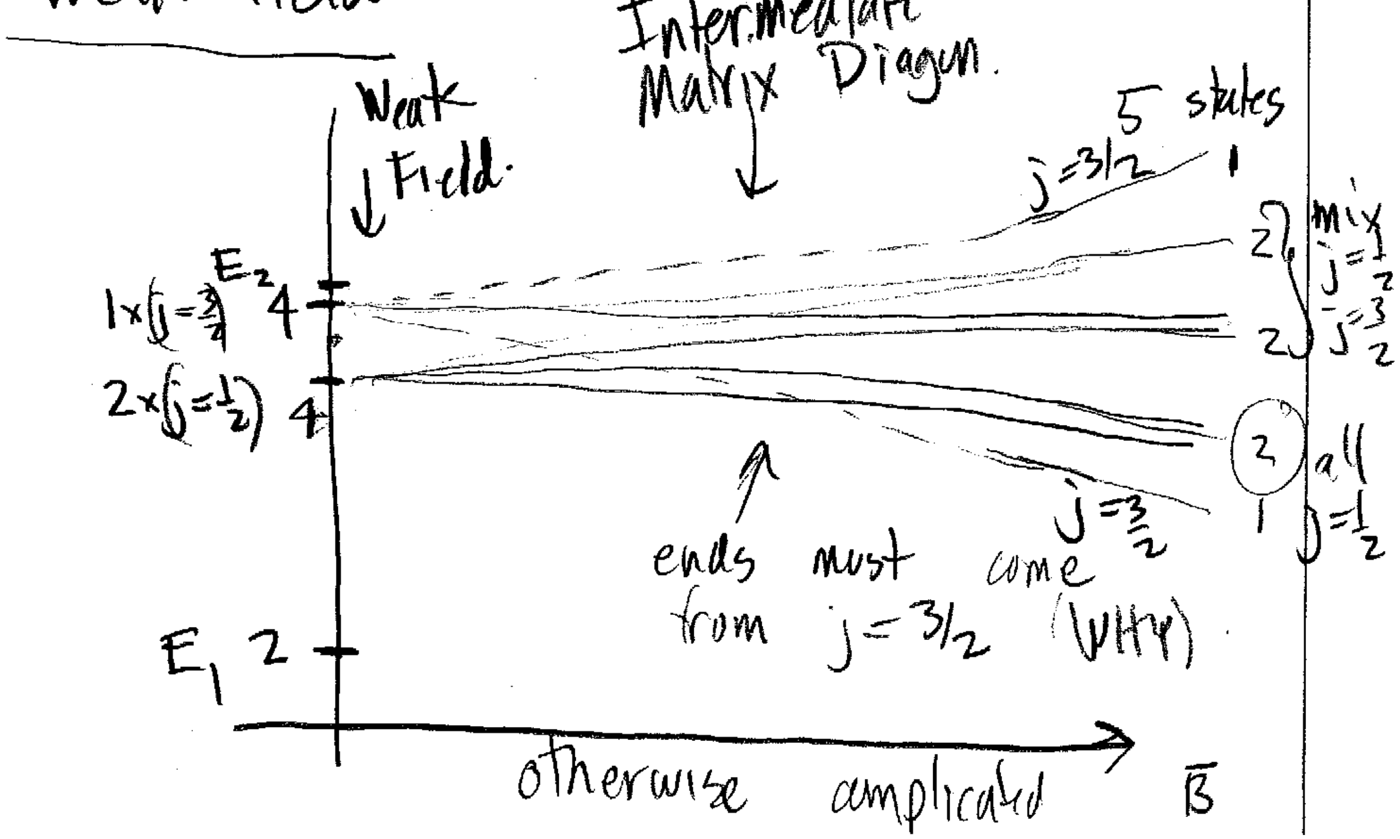
$L_z + 2S_z$	$m_\ell m_s$	
2	$+1/2$	$\{1 \cdot \frac{1}{2}\}$
1	0	$\{0 \cdot \frac{1}{2}$ twice$\}$
0	$-1/2$	$\{1 \cdot (-\frac{1}{2})$ or $-1(\frac{1}{2})\}$
-1	0	$\{0 \cdot -\frac{1}{2}$ twice$\}$
-2	$+1/2$	$\{-1 \cdot (-\frac{1}{2})$

NO SPLITTING... but 3/5 do shift...

include $H = \left(\frac{e^2}{4\pi\varepsilon_0}\right) \frac{1}{m^2c^2r^3} \vec{S} \cdot \vec{L}$

$$E^1_{fs,z} = \frac{13.6\ eV}{n^3} \alpha^2 \left[\frac{3}{4n} - \left[\frac{\ell(\ell+1) - m_\ell m_s}{\ell(\ell+1/2)(\ell+1)} \right] \right]$$

Weak field



Weak Field...

$E_z^1 = \langle n l j m_j | H_z' | n l j m_j \rangle$ why is l included? (parentage)

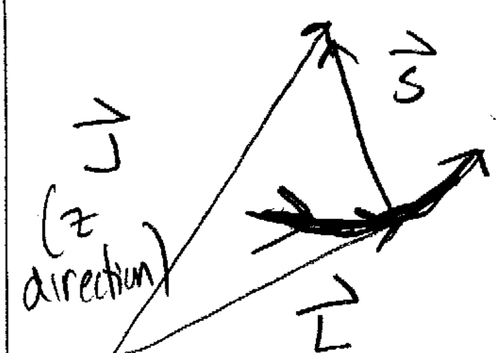
$H_z' = \frac{e}{2m} \vec{B}_{ext} \cdot \underbrace{(\vec{L} + 2\vec{S})}_{\text{expectation value is hard...}}$

$\vec{L} + 2\vec{S} = \vec{J} + \vec{S}$

- Take z-direction along $\vec{B}_{ext}$, so $\vec{B}_{ext} \cdot \vec{J} = B_{ext} \cdot J_z$
- But, $\langle n l j m_j | B_{ext} S_z | n l j m_j \rangle$ is not easy... why? $|n l j m_j\rangle$ is a combination of spin up + spin down.
- "Physical Trick"

$\vec{L} + \vec{S}$ precessing

$\vec{J}$ constant



$$\langle S_z \rangle = \frac{\langle \vec{S} \cdot \vec{J} \rangle \langle \vec{J} \rangle}{\langle J^2 \rangle}$$

$$\langle S_z \rangle = \frac{\langle \vec{S} \cdot \vec{J} \rangle m_j \hbar}{\hbar^2 j(j+1)}$$

$$\vec{L} = \vec{J} - \vec{S}$$

$$L^2 = J^2 + S^2 - 2\vec{S}\cdot\vec{J}$$

$$\langle \vec{S}\cdot\vec{J} \rangle = \frac{1}{2}(J^2 + S^2 - L^2)$$

$$\langle S_z \rangle = \left[\frac{j(j+1) + \frac{3}{4} - l(l+1)}{2j(j+1)}\right](m_j \hbar)$$

And so ...

$$\left\langle \vec{B}_{ext}\cdot(\vec{L} + 2\vec{S}) \right\rangle = B_{ext}\underbrace{\left[1 + \frac{j(j+1) - l(l+1) + \frac{3}{4}}{2j(j+1)}\right]}_{\text{called the "Landé g-factor" } g_j} m_j \hbar$$

example:

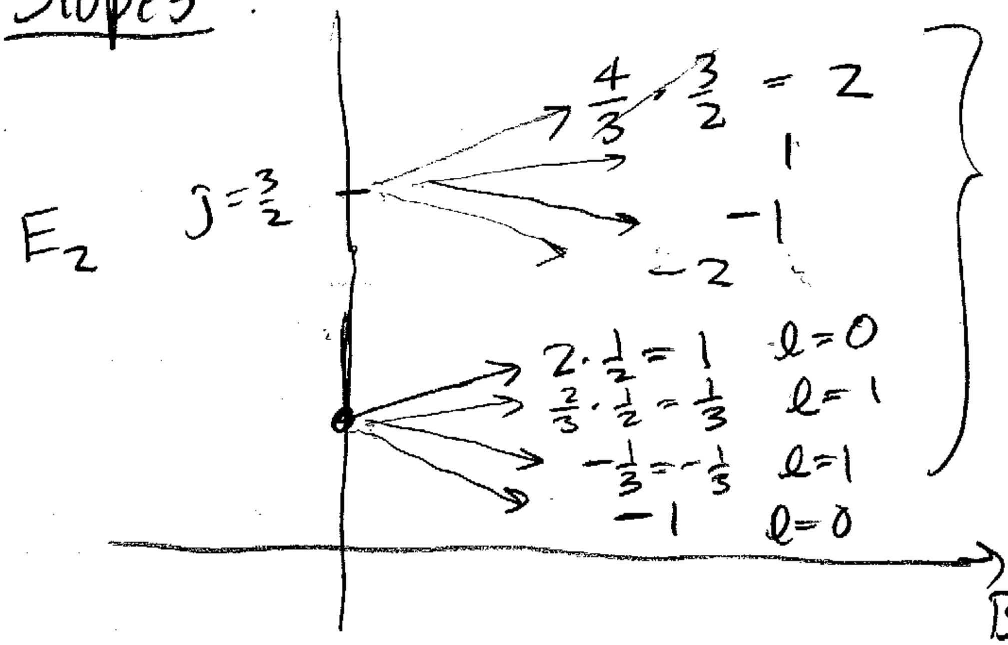
parentheses matters

$l=0,\ j=\frac{1}{2}$ $\quad g_{\frac{1}{2}} = 1 + \dfrac{\frac{3}{4} - 0 + \frac{3}{4}}{2\cdot\frac{1}{2}\left(\frac{3}{2}\right)} = 2$

$l=1,\ j=\frac{1}{2}$ $\quad g_{\frac{1}{2}} = 1 + \dfrac{\frac{3}{4} - 2 + \frac{3}{4}}{2\cdot\frac{1}{2}\left(\frac{3}{2}\right)} = 1 + \dfrac{-\frac{1}{2}}{\frac{3}{2}} = \dfrac{2}{3}$

$l=1,\ j=\frac{3}{2}$ $\quad g_{\frac{3}{2}} = 1 + \dfrac{\frac{3}{2}\cdot\frac{5}{2} - 2 + \frac{3}{4}}{2\cdot\frac{15}{4}} = \dfrac{4}{3}$

Slopes:



connecting to strong field complex!

$\boxed{8 \downarrow 5}$