$$\underbrace{E_m^0 \langle \psi_m^0 | \psi_n^1 \rangle}_{\text{study}} + \underbrace{\langle \psi_m^0 | H' | \psi_n^0 \rangle}_{\text{compute}} = \underbrace{E_n^0 \langle \psi_m^0 | \psi_n^1 \rangle}_{\text{study}} \quad m \neq n$$

but $|\psi_n^1\rangle = \underbrace{\sum_m |\psi_m^0\rangle \langle \psi_m^0 | \psi_n^1 \rangle}$

completeness means

$$\sum_m |\psi_m^0\rangle\langle\psi_m^0| = \underset{\sim}{\mathbb{1}}$$

another viewpoint...

$$|\psi_n^1\rangle = \sum_\ell c_\ell^{(n)} |\psi_\ell^0\rangle$$

$$\langle \psi_m^0 | \psi_n^1 \rangle = \underbrace{\sum_\ell c_\ell^{(n)} \underbrace{\langle \psi_m^0 | \psi_\ell^0 \rangle}_{\delta_{m\ell}}}$$

$$\langle \psi_m^0 | \psi_n^1 \rangle = c_m^{(n)}$$

so

$$(E_n^0 - E_m^0) \langle \psi_m^0 | \psi_n^1 \rangle = \langle \psi_m^0 | H' | \psi_n^0 \rangle$$

$m \neq n$

$$\boxed{c_m^{(n)} = \langle \psi_m^0 | \psi_1^0 \rangle = \frac{\langle \psi_m^0 | H' | \psi_n^0 \rangle}{E_n^0 - E_m^0}}$$

$m \neq n$

what about $c_n^{(n)}$? $= \langle \psi_n^0 | \psi_n^1 \rangle$

$\langle \psi_n^0 | \psi_n^1 \rangle$ dropped out of E_n^1 !

$\langle \psi_n^0 | \psi_n^1 \rangle$ is unconstrained to first order... why not set it $= 0$? SIMPLICITY

Getting then that

$$\boxed{|\psi_n^1\rangle = \sum_{m \neq n} |\psi_m^0\rangle \frac{\langle \psi_m^0 | H' | \psi_n^0 \rangle}{E_n^0 - E_m^0}}$$

$E_n^0 \neq E_m^0$ or else failure

$\Rightarrow$ NON-DEGENERATE ONLY

6.2 (b)

$\langle \psi_m^0 | H' | \psi_n^0 \rangle \neq 0$ only when m + n are both odd.

$\alpha\,\delta(x - \frac{a}{2})$

(princess + pea)

$$\int_0^a dx \sqrt{\frac{2}{a}} \sin\left(\frac{\pi m}{a} x\right) \alpha\,\delta\left(x - \frac{a}{2}\right) \sqrt{\frac{2}{a}} \sin\left(\frac{\pi n}{a} x\right)$$

$= 0$ if either m, n or both even

$= \frac{2}{a}\alpha$ both m + n odd

$$E_n^0 - E_m^0 = (n^2 - m^2)\frac{\pi^2 \hbar^2}{2ma^2}$$

$$C_3^{(1)} = \frac{\cancel{4}ma\alpha}{-\cancel{8}\pi^2\hbar^2} \qquad C_5^{(1)} = -\frac{\cancel{4}ma\alpha}{\cancel{24}\pi^2\hbar^2}$$

2 6

$$C_7^{(1)} = -\frac{\cancel{4}ma\alpha}{\cancel{48}\pi^2\hbar^2} \quad \text{etc}$$

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Second Order Energy Shift

First order : expectation value of the perturbation.

$$E_n^1 = \langle \psi_n^0 | H' | \psi_n^0 \rangle$$

when H^0 symmetric, ψ_n^0 are even or odd under "Parity" (Reflection about symmetry axis).

H' : even , $E_n^1 \neq 0$

odd , $E_n^1 = 0$

Odd H' contributes to: E_n^2 .

To get there, look at λ^2 and project on to $\langle \psi_n^0 |$

$$\langle \psi_n^0 | H^0 | \psi_n^2 \rangle + \langle \psi_n^0 | H' | \psi_n^1 \rangle = E_n^0 \langle \psi_n^0 | \psi_n^2 \rangle + E_n^1 \langle \psi_n^0 | \psi_n^1 \rangle + E_n^2 \cancelto{1}{\langle \psi_n^0 | \psi_n^0 \rangle}$$

$\langle \psi_n^0 | \psi_n^1 \rangle = 0$ by choice ....

$$E_n^2 = \langle \psi_n^0 | H' | \psi_n^1 \rangle$$

want totally in terms of unperturbed stuff

$$E_n^2 = \langle \psi_n^0 | \left\{ \sum_{m \neq n} \frac{H' | \psi_m^0 \rangle \langle \psi_m^0 | H' | \psi_n^0 \rangle}{E_n^0 - E_m^0} \right\}$$

$$\langle \psi_n^0 | H' | \psi_m^0 \rangle = \langle \psi_m^0 | H'^{\dagger} | \psi_n^0 \rangle^*$$

$H'^{\dagger} = H'$ Hermitian
Real e.v.'s

$$= \langle \psi_m^0 | H' | \psi_n^0 \rangle^*$$

and so

$$\boxed{E_n^2 = \sum_{m \neq n} \frac{|\langle \psi_m^0 | H' | \psi_n^0 \rangle|^2}{E_n^0 - E_m^0}}$$

$$= \sum_{m \neq n} \underbrace{\frac{|\langle \psi_m^0 | H' | \psi_n^0 \rangle|}{E_n^0 - E_m^0}} \; \underbrace{|\langle \psi_m^0 | H' | \psi_n^0 \rangle|}$$

"Penalty" factor for getting that energy, must $\ll 1$

NO DEGENERACY

"Energy acquired $m \to n$ transition caused by H'"

(H^0 doesn't do, $\langle \psi_m^0 | H^0 | \psi_n^0 \rangle = E_n \langle \psi_m^0 | \psi_n^0 \rangle = 0 \quad m \neq n$)

$\boxed{6.4(a)}$ $H' = \alpha\delta(x - a/2)$

$\langle\psi_m^0|H'|\psi_n^0\rangle = 0$ if either m or n is even, since $\psi_n^0(a/2) = 0$ when n = even

note $m \neq n$ in sum

$= \frac{2}{a}\alpha$ $m \neq n$ and both odd.

$$E_n^0 = \frac{n^2\pi^2\hbar^2}{2ma^2}$$

so, $$E_n^2 = \left(\frac{2}{a}\alpha\right)^2 \frac{1}{\frac{\pi^2\hbar^2}{2ma^2}} \sum_{\substack{m\neq n \\ \text{both odd}}} \frac{1}{n^2 - m^2}$$

$$= \frac{8m\alpha^2}{\pi^2\hbar^2} \underbrace{\sum_{\substack{m\neq n \\ \text{both odd}}} \frac{1}{n^2 - m^2}}_{\text{apparently (no proof here)}}$$

$$= -\frac{1}{4n^2}$$

Could get ψ_n^2

Betcha $E_n^3 = \langle\psi_n^0|H'|\psi_n^2\rangle$

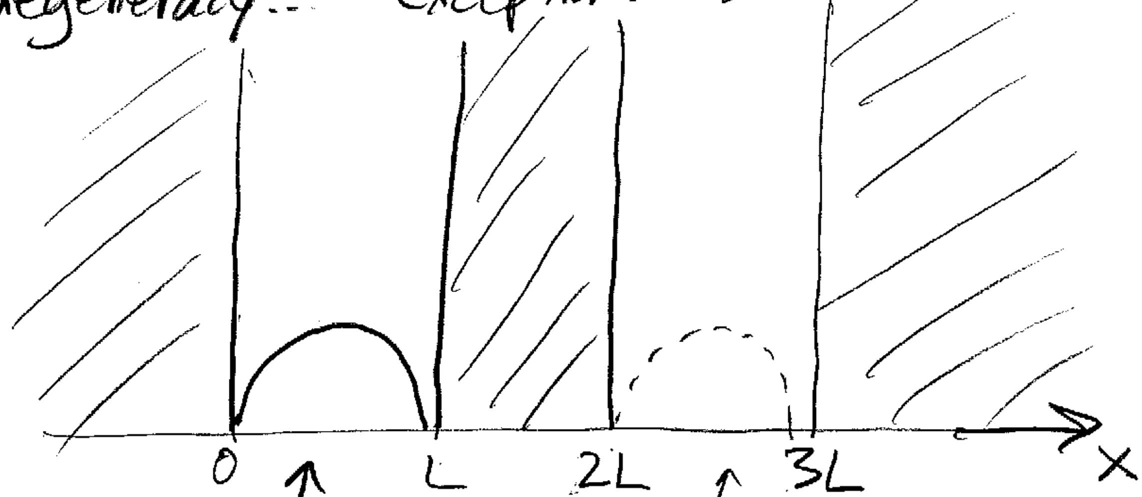
<u>THINK</u>: $[H^0, H'] = 0$ no need for perturbation theory
Simultaneous E.V.

$[H^0, H'] \neq 0$, $\langle\psi_m^0|H'|\psi_n^0\rangle \neq 0$, $m \neq n$

Degeneracy (a little different than text, pp. 257-266).

→ when ≥ 2 distinct states share an eigenvalue of H^o

→ most simple 1-d $V(x)$ don't have degeneracy... exception: "Double Well"



$\psi_a = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi x}{L}\right)$ $0 < x < L$

$= 0$ elsewhere

$\psi_b = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi(x-2L)}{L}\right)$ $2L < x < 3L$

$= 0$ elsewhere

Both have energies $\frac{\hbar^2\pi^2}{2mL^2}$

"Energy Denominator" in ψ' <u>blows up</u>

→ don't worry, it just reverts back to "regular QM"

<u>Note</u>: $\alpha\psi_a + \beta\psi_b$ also an eigenstate of H^o

Now imagine a perturbation, H'.

First + Most Important Question...
What does the matrix of H' look like in the "degenerate subspace"?

Two possibilities.

① Matrix is diagonal... 2x2 version

$$\begin{pmatrix} \langle \psi_a^0 | H' | \psi_a^0 \rangle & \langle \psi_a^0 | H' | \psi_b^0 \rangle \\ \langle \psi_b^0 | H' | \psi_a^0 \rangle & \langle \psi_b^0 | H' | \psi_b^0 \rangle \end{pmatrix} \equiv \begin{pmatrix} W_{aa} & W_{ab} \\ W_{ba} & W_{bb} \end{pmatrix}$$

$\Rightarrow$ definitely Hermitian.. $W_{ba}^* = W_{ab}$

$\Rightarrow$ Diagonal... $W_{ab} = W_{ba} = 0$!

$\Rightarrow$ means, original basis is OK! "GOOD"

Example: $\alpha\delta(x - a/2)$ only left side of double well.

$\Rightarrow \frac{p^4}{8m^3c^2}$ ... Spherical perturbation.

$\Rightarrow E_a^1 = W_{aa} \qquad E_b^1 = W_{bb}$
just like NON-DEGENERATE

$\Rightarrow [H^0, H'] = 0$

② Matrix NOT Diagonal... $W_{ab} \neq 0$

e.v. = those of 2×2 matrix.

$$\begin{vmatrix} W_{aa}-\lambda & W_{ab} \\ W_{ba} & W_{bb}-\lambda \end{vmatrix} = 0$$

$$(W_{aa}-\lambda)(W_{bb}-\lambda) - |W_{ab}|^2 = 0$$

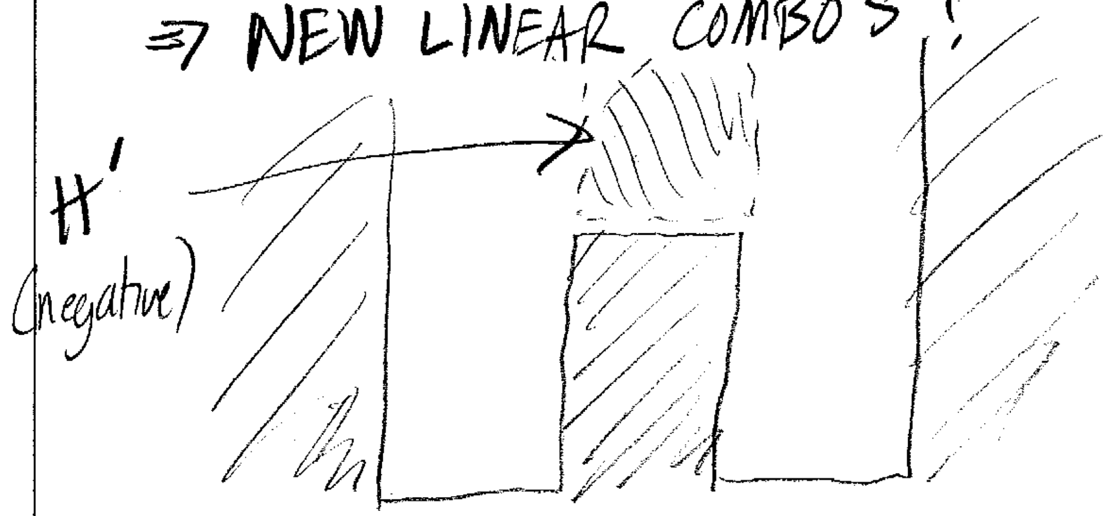
$$\lambda^2 - (W_{aa}+W_{bb})\lambda + W_{aa}W_{bb} - |W_{ab}|^2 = 0$$

$$\lambda = \frac{1}{2}\left[(W_{aa}+W_{bb}) \pm \sqrt{(W_{aa}-W_{bb})^2 + 4|W_{ab}|^2}\right]$$

⇒ Just plain old QM

⇒ Original basis not "good"

⇒ NEW LINEAR COMBOS!



Symmetry: New combos:

$$\frac{1}{\sqrt{2}}\left[\psi_a \pm \psi_b\right]$$