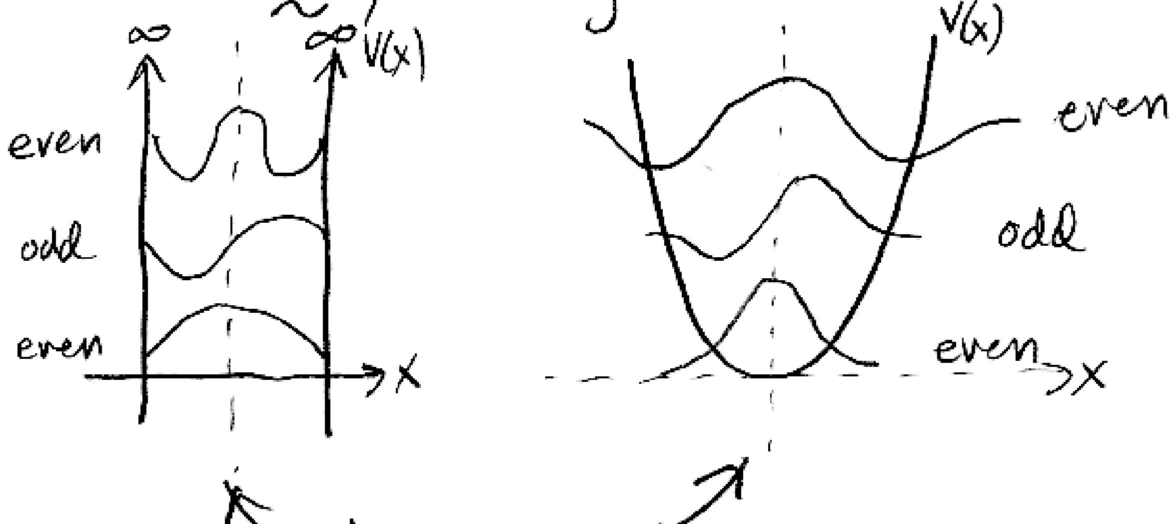
Consequences :

1) Simultaneous eigenstates of $\underset{\sim}{H}$ and $\underset{\sim}{\Pi}$, meaning:



both invarient under parity

so, eigenfunctions end up being eigenstates of parity

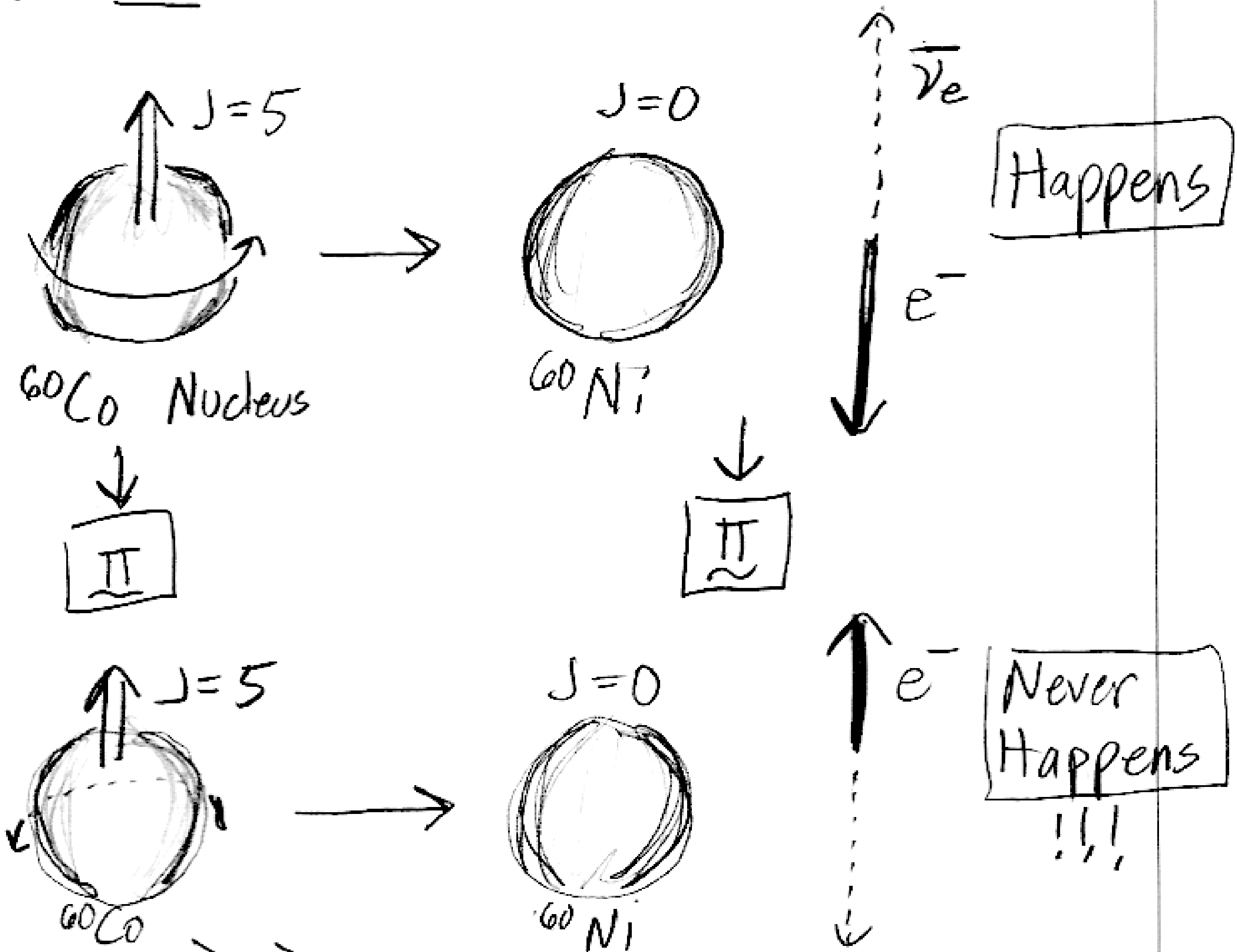
2) Ehrenfest: $\frac{d\langle \underset{\sim}{\Pi} \rangle}{dt} = \langle [\underset{\sim}{\Pi}, \underset{\sim}{H}] \rangle$

when

$\frac{d\langle \Pi \rangle}{dt} = 0$ then

Meaning: once in an eigenstate of parity, will be so forevermore

There are fundamental Hamiltonians that are <u>not</u> invariant



1) $\vec{J} \propto \vec{x} \times \vec{p}$

but $(-\vec{x}) \times (-\vec{p})$

$= \vec{x} \times \vec{p}$

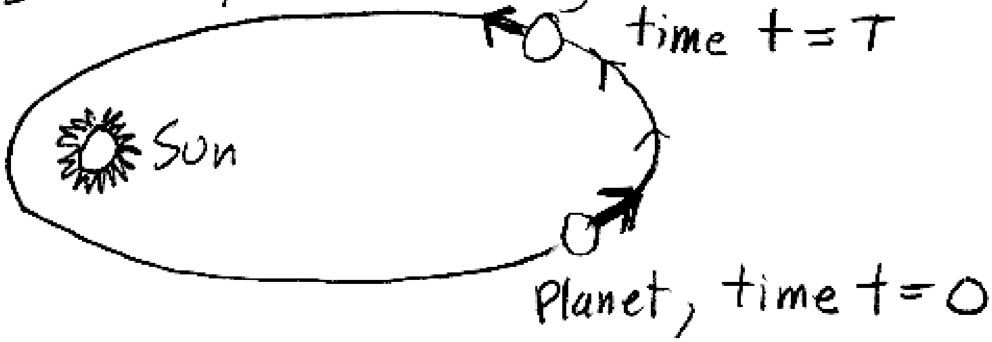
no change!

2) visualize motion that makes $\vec{J}$

The occurrence of the process on the top; absence of the process on the bottom is the evidence that the "Weak Interaction"

Time Reversal Symmetry

Textbook's motivation is quite good
Imagine a planet orbiting the son....



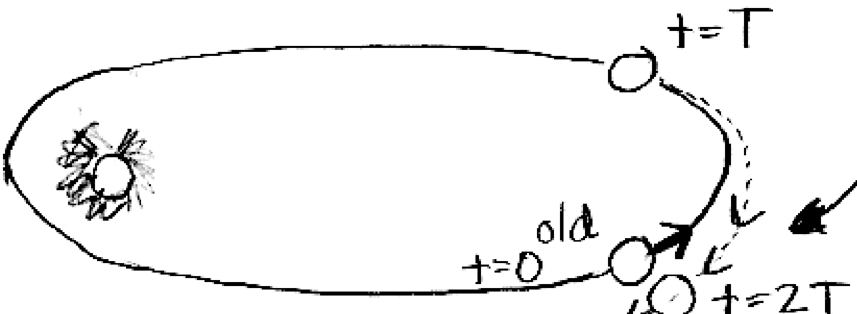
Now suppose, at $t=T$, someone (you, me, superman, a more powerful deity) reverses the direction of $\vec{p}$, but leaves $\vec{x}$, the position, unchanged. This is like reversing the direction of the arrow of time, since:

$$\vec{p} = m\dot{\vec{x}} = m\frac{d}{dt}\vec{x}$$

reverse t: $t' = -t$

$$\vec{p}\,' = m\frac{d}{dt'}\vec{x} = m\frac{d\vec{x}}{dt}\frac{dt}{dt'} = -m\frac{d\vec{x}}{dt} = -\vec{p}$$

Now let the planet move, subject to the forces/Hamiltonian that was previously present prior to the time reversal step.



when the planet does not return to the same spot, the underlying Hamiltoni is not Time Reversa Invariant (TRI) when the planet satisfies:

$$\left.\begin{aligned}\vec{x}(0) &= \vec{x}(2T) \\ \vec{p}(0) &= -\vec{p}(2T)\end{aligned}\right\}$$ underlying Hamiltonian $\underline{\underline{\text{is}}}$ Time Reversal Invariant (TRI)

Math:

$$x_r(t) \equiv x(-t)$$

if $m\dfrac{d^2x(t)}{dt^2} = F(x(t))$ depends only on position

then does $m\dfrac{d^2x_r(t)}{dt^2} = F(x_r(t))$?

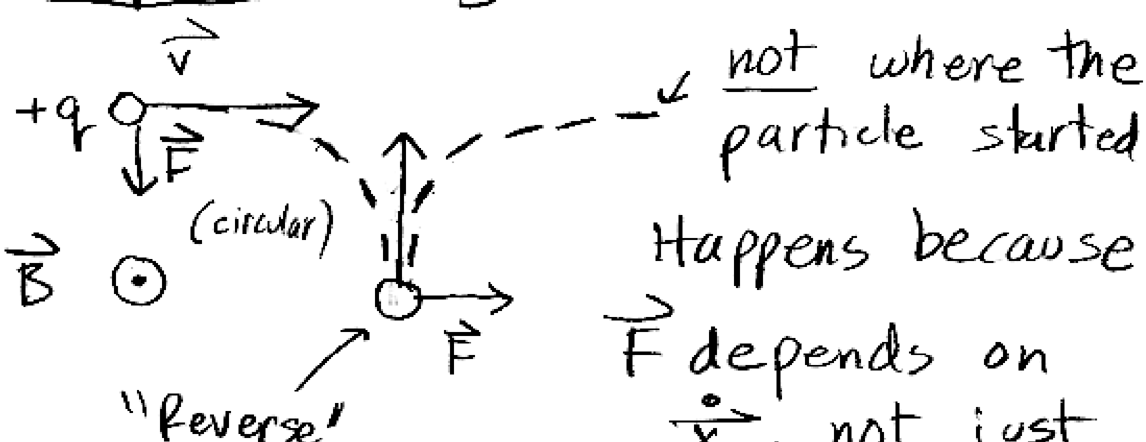
$$m\frac{d^2x_r(t)}{dt^2} = m\frac{d^2x(-t)}{dt^2} = m\frac{d^2x(-t)}{d(-t)^2}\cdot\left(\frac{d(-t)}{dt}\right)\left(\frac{d(-t)}{dt}\right)$$

$$= m\frac{d^2x(-t)}{d(-t)^2}\times(-1)^2 = m\frac{d^2x(-t)}{d(-t)^2} = F(x(-t))$$

$$= F(x_r(t))$$

so, $m\dfrac{d^2x_r(t)}{dt^2} = F(x_r(t))$

An exception: magnetic fields.



Happens because $\vec{F}$ depends on $\dot{\vec{x}}$, not just $\vec{x}$.

However, if particles that generate $\vec{B}$ are also subjected to time reversal, then $\vec{B}$ will also change sign, and the system becomes TRI.

Translation to Quantum Mechanics

$\psi(x,t)$ can be decomposed into plane waves, of the form

$$e^{\frac{i}{\hbar}(px - Et)}$$

An operation that takes $x \to x$, $p \to -p$, $t \to -t$ is complex conjugation

so

"TR wavefunction" conjugate.

$$\psi(x,0) \qquad\qquad \psi^*(x,0)$$

evolve to $t = T$ (constant $\underset{\sim}{H}$)

$$e^{-\frac{i}{\hbar}T\underset{\sim}{H}}\psi(x,0)$$

conjugate

$$e^{+\frac{iT\underset{\sim}{H}^*}{\hbar}}\psi^*(x,0)$$

evolve forward in time from T to 2T

$$e^{-\frac{iT\underset{\sim}{H}}{\hbar}}e^{+\frac{iT\underset{\sim}{H}^*}{\hbar}}\psi^*(x,0) \overset{?}{=} \psi^*(x,0)$$

what condition on $\underset{\sim}{H}$ satisfies?

when $\underset{\sim}{H} = \underset{\sim}{H}^*$

then $e^{-\frac{iT\underset{\sim}{H}}{\hbar}}e^{\frac{iTH^*}{\hbar}} = \underset{\sim}{\mathbb{1}}$

$\underset{\sim}{H}$ is time-reversal invariant in quantum mechanics when $\underset{\sim}{H} = \underset{\sim}{H}^*$; this is <u>not</u> the same, necessarily, as $\underset{\sim}{H} = \underset{\sim}{H}^\dagger$