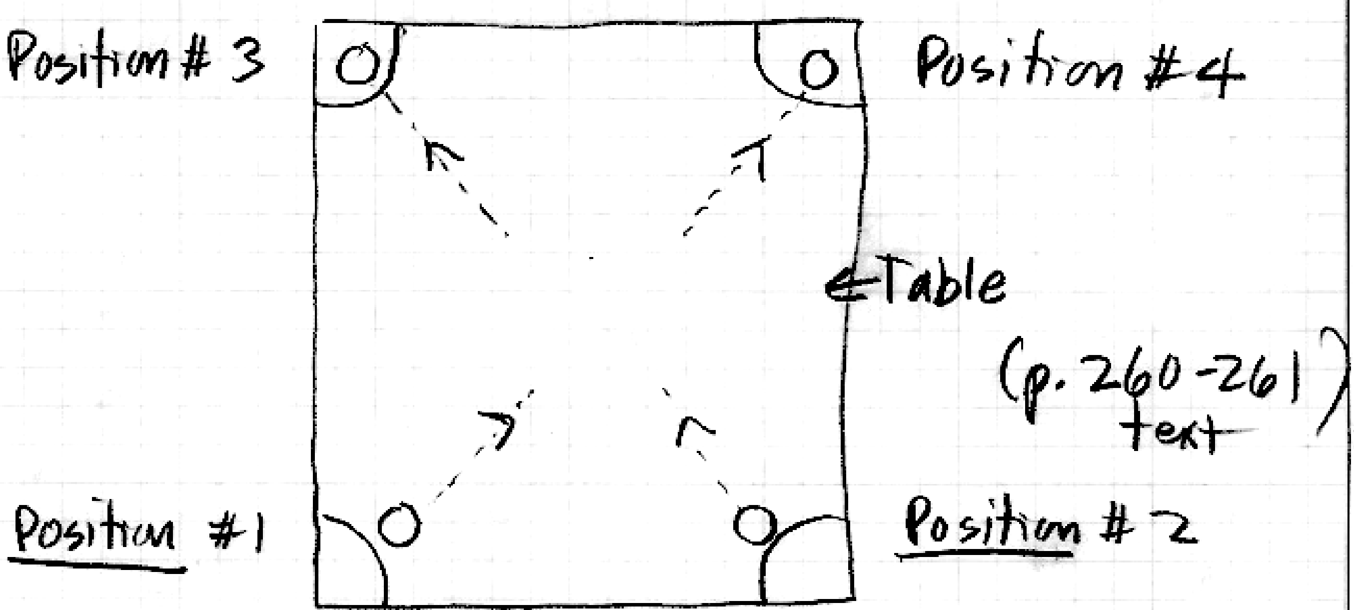
Identical Particles

Concepts one of the "quiet revolutions" of quantum physics

Classical Pool-Table Example

Two "truly indistinguishable" cue-balls (actually, this is a fallacy.. cue-balls can always be "marked").



Heart of the matter:

can you tell whether ball at position #1 ends up at position #3 or position #4 ?

Classically: yes, for two reasons

(a) can "follow" the trajectories of the balls.

(b) can "label" the balls.

Quantum Mechanically:

(a) "following" implies new measurements, and these disturb the state! So an experiment with "following" is not the same thing

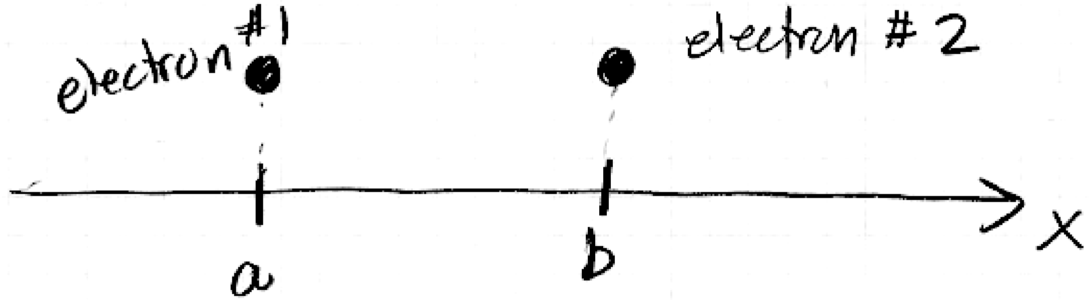
(b) cannot "label" the balls, if truly identical.

⇒ really true; cannot "label" electrons, protons, etc

⇒ these particles are not "tiny universes" .. which would be distinguishable...

Feynman... maybe there is only one electron, that zips around... explains "identicality"

Symmetric and Antisymmetric.



Electron #1 at 1-d position $x=a$
#2 at 1-d position $x=b$

Cannot restrict measurement apparatus to just #1 or #2; experimentally just measure <u>electrons</u>.

Measuring $\underset{\sim}{x}$ of each electron, you'll get $a + b$. You won't know whether a goes with #1 or #2; similarly for b.

$$|\psi(a,b)\rangle = \alpha\, |\psi(b,a)\rangle$$

state with #1's $x = a$, #2's $x = b$

any constant

state with #2's $x = a$ #1's $x = b$

This relationship follows from the concept that the "direction" of a ket or abstract vector is what has the physical meaning; the magnitude is chosen for convenience (usually normalized = 1). Since we cannot distinguish which is "right", $|\psi(a,b)\rangle$ and $|\psi(b,a)\rangle$ must be the "same"; or proportional.

$|ab\rangle$ and $|ba\rangle$ are candidates for $|\psi(a,b)\rangle$

$|ab\rangle$: #1 $x = a$, #2 $x = b$

$|ba\rangle$: #1 $x = b$, #2 $x = a$

most generally, though:

$$|\psi(a,b)\rangle = \underbrace{\beta|ab\rangle + \gamma|ba\rangle}$$

note this is an eigenstate of $\underset{\sim}{X}_1 + \underset{\sim}{X}_2$ ... which has a whole degenerate subspace of eigenstates

Must be:

$$|\psi(a,b)\rangle = \alpha|\psi(b,a)\rangle$$

$$\beta|ab\rangle + \gamma|ba\rangle = \alpha\left[\beta|ba\rangle + \gamma|ab\rangle\right]$$

... claim $\langle ba|ab\rangle = \langle ab|ba\rangle = 0$

and so: (assuming $a \neq b$)

$$\beta = \alpha\gamma \qquad \gamma = \alpha\beta$$

$$\gamma = \alpha^2\cancel{\gamma}$$

$$\alpha^2 = 1$$

$$\boxed{\alpha = \pm 1}$$

$$\boxed{\beta = \pm\gamma}$$

Symmetric state

$$|ab, S\rangle = |ab\rangle + |ba\rangle \quad \text{(un-normalized)}$$

$$|ab, A\rangle = |ab\rangle - |ba\rangle \quad (\quad " \quad)$$

Nature:

Some particles (called Bosons) ALWAYS occupy symmetric states:

pair bosons... $|\omega_1 \omega_2\rangle + |\omega_2 \omega_1\rangle$

eigenvalues of $\underset{\sim}{\Omega}_1 + \underset{\sim}{\Omega}_2$ (generalized from $\underset{\sim}{X}$

bosons have intrinsic (spin) angular momentum $= 0, \hbar, 2\hbar, \ldots$ (even multiple of $\frac{\hbar}{2}$)

Examples of Bosons:

photon (γ) spin $= \hbar$ (gluons $W^{\pm}, Z^0$ too).

^{4}He atom spin $= 0$

Higgs' (H^0) spin $= 0$ pion: $\pi = \begin{pmatrix} u\bar{d} \\ u\bar{u} \\ d\bar{d} \\ \bar{u}d \end{pmatrix}$

Other particles (known as Fermions) ALWAYS

pair fermions... $|\omega_1 \omega_2\rangle - |\omega_2 \omega_1\rangle$

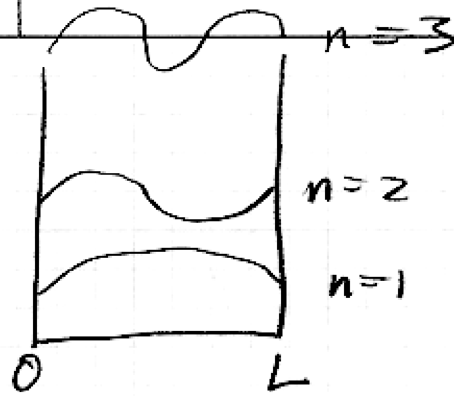
spin ... $\hbar/2, 3\hbar/2, \ldots$ (odd multiples of $\hbar/2$).

Examples:

electron (e) proton (p) neutron (n)

^{3}He atom spin $= \hbar/2$

quarks + leptons.



Concrete

Visualize ω_1, ω_2 ....

$\uparrow$ n_1 $\quad \uparrow$ n_2

... drop identical particles into a square well.

$$E_n = \frac{\hbar^2 \pi^2}{2mL^2} n^2$$

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right).$$

Consider $|11\rangle$ $\leftarrow$ idea is both particles dropped into ground state

when particles identical...

BOSONS: $|11, S\rangle \propto |11\rangle + |11\rangle = 2|11\rangle$

$$|11, S\rangle = |11\rangle$$

FERMIONS: $|11, A\rangle \propto |11\rangle - |11\rangle = 0$!

The Pauli Principle : Identical fermions are never in identical quantum states!

why... (?) except spin $\leftrightarrow$ statistics connection.