In #1 space: In #2 space:

$$\underset{\sim}{\Omega}_1^{(1)} \doteq \begin{array}{c|cc} & |+\rangle & |-\rangle \\ \langle +| & a & b \\ \langle -| & c & d \end{array} \qquad \underset{\sim}{\Omega}_2^{(2)} \doteq \begin{array}{c|cc} & |+\rangle & |-\rangle \\ \langle +| & e & f \\ \langle -| & g & h \end{array}$$

What is representation of :

(#1) $\underset{\sim}{\Omega}_1^{(1)\otimes(2)}$ = $\underset{\sim}{\Omega}_1$ in #1 space

$\underset{\sim}{1}$ in #2 space

$$= \underset{\sim}{\Omega}_1^{(1)} \otimes \underset{\sim}{1}^{(2)}$$

in the basis of the 4 product states?

→ complexity merely from the ordering of the product states.

$$\langle ++| \underset{\sim}{\Omega}^{(1)} \otimes \underset{\sim}{1}^{(2)} |++\rangle = \underbrace{\langle +|\underset{\sim}{\Omega}^{(1)}|+\rangle}_{a} \underbrace{\langle +|\underset{\sim}{1}^{(2)}|+\rangle}_{1}$$

$$= a$$

$$\langle ++| \underset{\sim}{\Omega}^{(1)} \otimes \underset{\sim}{1}^{(2)} |+-\rangle = \underbrace{\langle +|\underset{\sim}{\Omega}^{(1)}|+\rangle}_{a} \underbrace{\langle +|\underset{\sim}{1}^{(2)}|-\rangle}_{0}$$

Rule: second states must be identical, and then transcribe appropriate elements of $\underset{\sim}{\Omega}_1^{(1)}$.

$\Omega_1^{(1)\otimes(2)}$	$\lvert ++\rangle$	$\lvert +-\rangle$	$\lvert -+\rangle$	$\lvert --\rangle$
$\langle ++\rvert$	a	0	b	0
$\langle +-\rvert$	0	a	0	b
$\langle -+\rvert$	c	0	d	0
$\langle --\rvert$	0	c	0	d

$\Omega_2^{(1)\otimes(2)}$: first states identical, transcribe $\underset{\sim}{\Omega}_2^{(1)}$

$\langle ++\rvert$	e	f	0	0
$\langle +-\rvert$	g	h	0	0
$\langle -+\rvert$	0	0	e	f
$\langle --\rvert$	0	0	g	h

$\underset{\sim}{\Omega}_1^{(1)} \otimes \Omega_2^{(2)}$

	$\lvert + + \rangle$	$\lvert + - \rangle$	$\lvert - + \rangle$	$\lvert - - \rangle$
$\langle + + \rvert$	$\langle + \lvert \underset{\sim}{\Omega}_1 \rvert + \rangle \cdot \langle + \lvert \underset{\sim}{\Omega}_2 \rvert + \rangle = a \cdot e$	$\langle + \lvert \underset{\sim}{\Omega}_1 \rvert + \rangle \langle + \lvert \underset{\sim}{\Omega}_2 \rvert - \rangle = a \cdot f$	$b \cdot e$	$b \cdot f$
$\langle + - \rvert$	$a \cdot g$	$a \cdot h$	$b \cdot g$	$b \cdot h$
$\langle - + \rvert$	$c \cdot e$	$c \cdot f$	$d \cdot e$	$d \cdot f$
$\langle - - \rvert$	$c \cdot g$	$c \cdot h$	$d \cdot g$	$d \cdot h$

Can also arrive at this result by simply multiplying the two matrices on p. 49.

Two Particles p. 254

most common Hamiltonian: (classical)

$$\frac{|\vec{p}_1|^2}{2m_1} + \frac{|\vec{p}_2|^2}{2m_2} + V(\vec{x}_1, \vec{x}_2)$$

→ omit y, z co-ordinates (1-dimensional)

two interesting cases:

(A) $V(x_1, x_2) = V_1(x_1) + V_2(x_2)$ ("separable")

(B) $\qquad = V(x_1 - x_2)$ ("2-body problem")

(A) Separable

$$\underset{\sim}{H} = \underbrace{\left(\frac{p_1^2}{2m_1} + V_1(\underset{\sim}{x_1})\right)}_{\#1 \text{ space } \underset{\sim}{H}_1} + \underbrace{\left(\frac{p_2^2}{2m_2} + V_2(\underset{\sim}{x_2})\right)}_{\#2 \text{ space } \underset{\sim}{H}_2}$$

product states are eigenstates

$$\underset{\sim}{H}_1 |E_1\rangle = E_1 |E_1\rangle \qquad \underset{\sim}{H}_2 |E_2\rangle = E_2 |E_2\rangle$$

then $|E_1\rangle|E_2\rangle = |E_1\rangle \otimes |E_2\rangle = |E_1 E_2\rangle$

is eigenstate of $\underset{\sim}{H} = \underset{\sim}{H}_1 + \underset{\sim}{H}_2$

$$(\underset{\sim}{H}_1 + \underset{\sim}{H}_2)|E_1 E_2\rangle = (E_1 + E_2)|E_1 E_2\rangle$$

Coordinate space:

$$\psi_{E_1+E_2}(x_1, x_2) = \psi_{E_1}(x_1)\,\psi_{E_2}(x_2)$$

$$\left[\frac{-\hbar^2}{2m_1}\frac{\partial^2}{\partial x_1^2} + V_1(x_1) - \frac{\hbar^2}{2m_2}\frac{\partial^2}{\partial x_2^2} + V_2(x_2)\right]\psi_{E_1}(x_1)\,\psi_{E_2}(x_2)$$

$$= \Bigg[\psi_{E_2}(x_2)\left\{\left(\frac{-\hbar^2}{2m_1}\frac{\partial^2}{\partial x_1^2} + V_1(x_1)\right)\psi_{E_1}(x_1)\right\}$$

$$+ \psi_{E_1}(x_1)\left\{\left(\frac{-\hbar^2}{2m_2}\frac{\partial^2}{\partial x_2^2} + V_2(x_2)\right)\psi_{E_2}(x_2)\right\}\Bigg]$$

$$= \left[\psi_{E_2}(x_2)\,E_1\,\psi_{E_1}(x_1) + \psi_{E_1}(x_1)\,E_2\,\psi_{E_2}(x_2)\right]$$

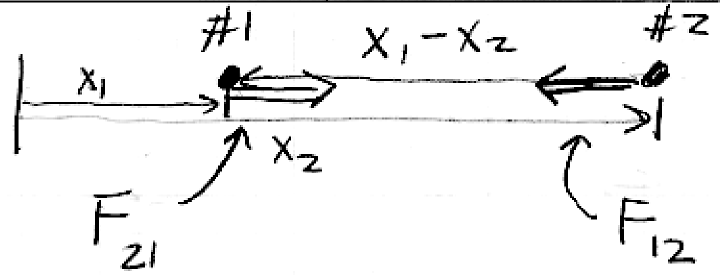
$$= (E_1 + E_2)\,\psi_{E_1}(x_1)\,\psi_{E_2}(x_2)$$

(B) "Two Body" $\Rightarrow$ not separable.

$\Rightarrow$ must change variables.

$V(x_1 - x_2)$; $\boxed{x = x_1 - x_2}$

other variable? Appeal to classical situation.... Newton's Third Law..



derived from $V(x_1 - x_2)$

Newton III: $F_{21} = -F_{12}$

$$\text{①} \quad m_1 \ddot{x}_1 = F_{21} = -F_{12}$$
$$\text{②} \quad m_2 \ddot{x}_2 = F_{12}$$

$$m_1 \ddot{x}_1 + m_2 \ddot{x}_2 = 0$$

$$\boxed{X = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}} \leftarrow \text{to get units right}$$

<u>Momenta</u>

conjugate to x:

$$\frac{\text{①}}{m_1} - \frac{\text{②}}{m_2} = \ddot{x}_1 - \ddot{x}_2 = \ddot{x} = \frac{F_{21}}{m_1} - \frac{F_{12}}{m_2}$$

$$\ddot{x} = \underbrace{\left(\frac{1}{m_1} + \frac{1}{m_2}\right)}_{\equiv \frac{1}{\mu}} F_{21} \qquad \mu = \frac{m_1 m_2}{m_1 + m_2}$$

$$\mu \ddot{x} = F_{21}$$

$$\boxed{p = \mu \dot{x} = \mu\left(\frac{p_1}{m_1} - \frac{p_2}{m_2}\right)}$$

conjugate to X:

$$\boxed{P = (m_1 + m_2)\dot{X} = p_1 + p_2}$$

Change of Variables?

Abstract ... check commutation relations.

$$[\underset{\sim}{x}, p] = [\underset{\sim}{x}_1 - \underset{\sim}{x}_2, \mu(\frac{p_1}{m_1} - \frac{p_2}{m_2})]$$

$$= \frac{\mu}{m_1}[\underset{\sim}{x}_1, p_1] + \frac{\mu}{m_2}[\underset{\sim}{x}_2, p_2]$$

$$= i\hbar(\frac{\mu}{m_1} + \frac{\mu}{m_2}) = i\hbar$$

Homework: $[\underset{\sim}{x}, \underset{\sim}{P}] = [\underset{\sim}{X}, p] = 0$

$[\underset{\sim}{X}, \underset{\sim}{P}] = i\hbar$

"proof" that variable change OK.

Representation:

$$i\hbar \frac{\partial}{\partial x} = i\hbar\mu(\frac{1}{m_1}\frac{\partial}{\partial x_1} - \frac{1}{m_2}\frac{\partial}{\partial x_2})$$

$$i\hbar \frac{\partial}{\partial X} = i\hbar(\frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_2})$$

really want to invert:

$$\frac{m_2}{\mu}\frac{\partial}{\partial x} = \frac{m_2}{m_1}\frac{\partial}{\partial x_1} - \frac{\partial}{\partial x_2}$$

$$\frac{\partial}{\partial X} = \frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_2}$$

$$\frac{\partial}{\partial X} + \frac{m_2}{\mu}\frac{\partial}{\partial x} = (1 + \frac{m_2}{m_1})\frac{\partial}{\partial x_1} = \frac{m_2}{\mu}\frac{\partial}{\partial x_1}$$

$$\frac{\partial}{\partial x_1} = \frac{\mu}{m_2}\frac{\partial}{\partial X} + \frac{\partial}{\partial x}$$

$$\frac{\partial}{\partial x_2} = \frac{\mu}{m_1}\frac{\partial}{\partial X} - \frac{\partial}{\partial x}$$

$$\frac{1}{m_1}\frac{\partial^2}{\partial x_1^2} + \frac{1}{m_2}\frac{\partial^2}{\partial x_2^2} = \frac{\mu^2}{m_1 m_2^2}\frac{\partial^2}{\partial X^2} + \frac{2\mu}{m_1 m_2}\frac{\partial^2}{\partial x \partial X} + \frac{1}{m_1}\frac{\partial^2}{\partial x^2}$$

$$+ \frac{\mu^2}{m_2 m_1^2}\frac{\partial^2}{\partial X^2} - \frac{2\mu}{m_1 m_2}\frac{\partial^2}{\partial x \partial X} + \frac{1}{m_2}\frac{\partial^2}{\partial x^2}$$

$$= \frac{\mu^2}{m_1 m_2}\underbrace{\left(\frac{1}{m_2} + \frac{1}{m_1}\right)}_{\mu}\frac{\partial^2}{\partial X^2} + \left(\frac{1}{m_1} + \frac{1}{m_2}\right)\frac{\partial^2}{\partial x^2}$$

$$= \frac{1}{\cancel{m_1 m_2}}\frac{\cancel{m_1 m_2}}{(m_1 + m_2)}\frac{\partial^2}{\partial X^2} + \frac{1}{\mu}\frac{\partial^2}{\partial x^2}$$

or

$$-\frac{\hbar^2}{2m_1}\frac{\partial^2}{\partial x_1^2} - \frac{\hbar^2}{2m_2}\frac{\partial^2}{\partial x_2^2} = \frac{-\hbar^2}{2(m_1+m_2)}\frac{\partial^2}{\partial X^2} - \frac{\hbar^2}{2\mu}\frac{\partial^2}{\partial x^2}$$

$$\boxed{\frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} = \frac{P^2}{2(m_1+m_2)} + \frac{p^2}{2\mu}}$$