Completion of argument about

$$u(y) = c_0\left[1 + \frac{(1-2\varepsilon)}{2}y^2 + \frac{(5-2\varepsilon)(1-2\varepsilon)}{4\cdot 3\cdot 2}y^4 + \frac{(9-2\varepsilon)(5-2\varepsilon)(1-2\varepsilon)}{6\cdot 5\cdot 4\cdot 3\cdot 2}y^6 + \ldots\right.$$

$$\left. + c_1\left\{y + \frac{(3-2\varepsilon)}{3\cdot 2}y^3 + \frac{(11-2\varepsilon)(7-2\varepsilon)(3-2\varepsilon)}{7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2}y^7 + \ldots\right.\right]$$

The most direct way to see the limiting value of this series is to look at:

$$\lim_{n\to\infty}\frac{c_{n+2}}{c_n} = \lim_{n\to\infty}\frac{2n+1-2\varepsilon}{(n+2)(n+1)} \to \frac{2}{n}$$

look at

$$y^m e^{y^2} = \sum_{k=0}^{\infty}\frac{y^{2k+m}}{k!}$$

$n = 2k+m$ so $k = \frac{1}{2}(n-m)$

so

$$\frac{c_{n+2}}{c_n} = \frac{\frac{1}{\left[\frac{1}{2}(n+2-m)\right]!}}{\frac{1}{\left[\frac{1}{2}(n-m)\right]!}} = \frac{\left[\frac{1}{2}(n-m)\right]!}{\left[\frac{1}{2}(n+2-m)\right]!}$$

but $\left[\frac{1}{2}(n+2-m)\right]! = \frac{1}{2}(n+2-m)\cdot\left[\frac{1}{2}(n-m)\right]!$

so $$\lim_{n\to\infty}\frac{c_{n+2}}{c_n} = \frac{2}{n+2-m} = \frac{2}{n}$$

so, $u(y) \to y^m e^{y^2}$ when series does not terminate