1. $\Psi(x,t) = N\, e^{i(k_0 x - \omega_0 t)}\left[1 + \cos\left(\frac{\Delta k}{2}x - \frac{\Delta\omega}{2}t\right)\right]$

a) $\Psi(x,0) = N\, e^{ik_0 x}\left[1 + \cos\left(\frac{\Delta k}{2}x\right)\right]$ (Choose N s.t. $\int dx |\Psi|^2 = 1$)

Find k_0, Δk from initial conditions

K.E. = 10eV : $K.E. = \frac{1}{2}mv^2 = \frac{p^2}{2m} \Rightarrow p = \sqrt{2m(K.E.)}$

$p = \hbar k \Rightarrow k_0 = \frac{1}{\hbar}\sqrt{2m_e(10eV)} \approx 1.7\cdot 10^{10}\ m^{-1}$

$\Delta(\hbar\omega) = 2eV$: $E = \frac{1}{2m}\hbar^2 k^2$ (Just $E = p^2/2m$)

$\Delta E = \frac{1}{m}\hbar^2 k \Delta k$ (Errors)

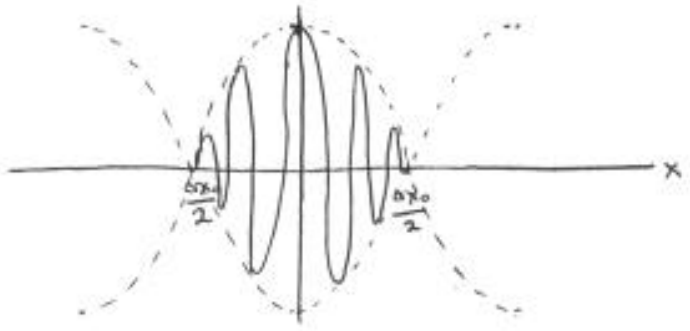
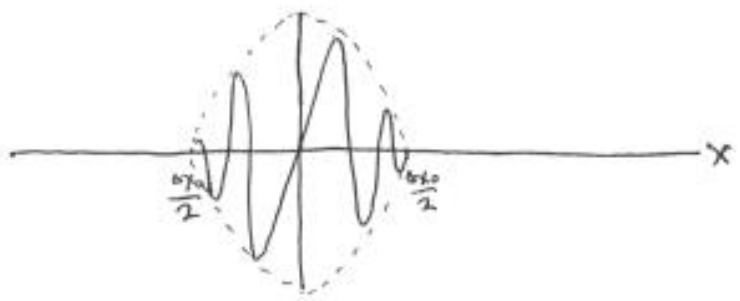
$\Rightarrow \Delta k = \frac{m_e}{\hbar^2 k_0}\Delta(\hbar\omega) \approx 1.7\cdot 10^{7}\ m^{-1}$

Need a def. for Δx. Let $\frac{1}{2}\Delta x$ be (dist. between max & min)/2

With $\Delta k \ll k_0$ we see that $e^{ik_0 x}$ part makes fast oscillations inside the envelope of the $\left[1 + \cos\frac{\Delta k}{2}x\right]$ part. We should choose Δx_0 based on the envelope fncn. $\left[1 + \cos\left(\frac{\Delta k}{2}x\right)\right]$ changes from its max w/ $x=0$ to its min when $\frac{\Delta k}{2}x = \pi$

$\Rightarrow \frac{\Delta k}{2}(\Delta x_0) = \pi \Rightarrow \Delta x_0 = \frac{2\pi}{\Delta k}$

Plot $\mathrm{Re}\,\Psi(x,0) \propto \cos(k_0 x)\left[1 + \cos\left(\frac{\Delta k}{2}x\right)\right]$



Plot $\mathrm{Im}\,\Psi(x,0) \propto \sin(k_0 x)\left[1 + \cos\left(\frac{\Delta k}{2}x\right)\right]$