UNIVERSITY OF CALIFORNIA, SANTA BARBARA
Department of Physics

Physics 115A
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FINAL EXAM (con't.)

2. A particle of mass m is in a potential $V = \frac{1}{2}m\omega^2 x^2$. The particle is in a state such that there is a probability of $\frac{1}{2}$ that a measurement of the energy will yield the value $\frac{\hbar\omega}{2}$ and a probability of $\frac{1}{2}$ the value $\frac{3}{2}\hbar\omega$.

(a) Is this a state of even, odd, or mixed parity?

$\psi = \frac{1}{\sqrt{2}}[u_0(x) + e^{i\theta}u_1(x)]$

from the equation page

$u_0(x)$ is even $u_1(x)$ is odd

Mixed Parity

(b) At the time $t = 0$ the expectation value of the momentum is $\langle p\rangle = \frac{\beta\hbar}{\sqrt{2}}$. Write an expression for the state vector in terms of the energy eigenstates.

$$\frac{\beta\hbar}{\sqrt{2}} = \frac{1}{2}\left[\left(\langle u_0| + e^{-i\theta}\langle u_1|\right) p\left(|u_0\rangle + e^{i\theta}|u_1\rangle\right)\right]$$

$$= \frac{1}{2}\left[\langle u_0|p|u_0\rangle + e^{-i\theta}\langle u_1|p|u_0\rangle + e^{i\theta}\langle u_0|p|u_1\rangle + \langle u_1|p|u_1\rangle\right]$$

$p = \frac{1}{\sqrt{2}i}(A - A^\dagger)p\hbar$ therefore $\langle u_0|p|u_0\rangle = \langle u_1|p|u_1\rangle = 0$

$$\frac{p\hbar}{\sqrt{2}} = \frac{1}{2\sqrt{2}i}p\hbar\left[-e^{-i\theta}\langle u_1|A^\dagger|u_0\rangle + e^{i\theta}\langle u_0|A|u_1\rangle\right]$$

$= \frac{p\hbar}{\sqrt{2}}\sin\theta$ so $\sin\theta = 1$, $\theta = \pi/2$

$e^{i\theta} = i$

$\psi = \frac{1}{\sqrt{2}}[u_0(x) + iu_1(x)]$