UNIVERSITY OF CALIFORNIA, SANTA BARBARA

Physics 115A — Department of Physics — Winter 1997

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ASSIGNMENT #8

Due Tuesday, March 11, 1997

1. Book Chapter 7, Problem 1.
2. Book Chapter 7, Problem 3.
3. Consider a perturbation to the simple harmonic oscillator potential,

$$H' = KX^3.$$

   (a) Express this in terms of the operators A and $A^\dagger$.

   (b) Let $|\phi_0\rangle$ be the ground state of the unperturbed simple harmonic oscillator. Evaluate the matrix elements $\langle\phi_0|\,H'\,|\phi_0\rangle$, $\langle\phi_1|\,H'\,|\phi_0\rangle$, and $\langle\phi_2|\,H'\,|\phi_0\rangle$.

4. Consider a harmonic oscillator of mass m and angular frequency ω. At time $t = 0$, the state of this oscillator is given by:

$$|\psi(0)\rangle = \sum_n c_n |u_n\rangle$$

   where the states $|u_n\rangle$ are stationary states with energies $(n + 1/2)\hbar\omega$.

   (a) What is the probability $\mathcal{P}$ that a measurement of the oscillator's energy performed at an arbitrary time $t > 0$, will yield a result greater than $2\hbar\omega$? When $\mathcal{P} = 0$, what are the non-zero coefficients c_n?

   (b) From now on, assume that only c_0 and c_1 are different from zero. Write the normalization condition for $|\psi(0)\rangle$ and the mean value $\langle H\rangle$ of the energy in terms of c_0 and c_1. With the additional requirement $\langle H\rangle = \hbar\omega$, calculate $|c_0|^2$ and $|c_1|^2$.

   (c) As the normalized state vector $|\psi(0)\rangle$ is defined only to within a global phase factor, we fix this factor by choosing c_0 real and positive. We set: $c_1 = |c_1|e^{i\theta_1}$. We assume that $\langle H\rangle = \hbar\omega$ and that:

$$\langle X\rangle = \tfrac{1}{2}\sqrt{\tfrac{\hbar}{m\omega}}.$$

   Calculate θ_1.

   (d) With $|\psi(0)\rangle$ so determined, write $|\psi(t)\rangle$ for $t > 0$ and calculate the value of θ_1 at t. Deduce the mean value $\langle X\rangle(t)$ of the position at t.