Physics 115A Ninth Problem Set

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Office Hour None this week!
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due Monday, March 10, 2003

1. In this problem find the propagator and interpret the results for the simplest 2-state system. Your understanding of this problem can be applied to Magnetic Resonant Imaging, neutrino oscillation, and CP violation, among other physics problems.

   For the 2-state system here, imagine that the eigenvalues are:

$$E_1 = +\frac{\hbar\omega}{2} \quad , \quad E_2 = -\frac{\hbar\omega}{2},$$

   where ω can be thought of simply as a parameter related to the energy eigenvalues. The propagator, $\mathbf{U}(t,0)$, is then:

$$\mathbf{U}(t,0) = |\hbar\omega/2\rangle e^{-i\omega t/2}\langle\hbar\omega/2| + |-\hbar\omega/2\rangle e^{i\omega t/2}\langle-\hbar\omega/2|$$

   (a) Find the matrix that represents the Hamiltonian, $\mathbf{H}$, in the eigenbasis of $\mathbf{H}$.

   (b) Find the matrix that represents the propagator $\mathbf{U}(t,0)$ in the eigenbasis of $\mathbf{H}$.

   (c) Use the matrix representations to evaluate the power series and show that:

$$e^{\frac{-i\mathbf{H}t}{\hbar}} = \mathbf{U}(t,0)$$

   (d) A system starts in the state $|\psi(0)\rangle$, which is represented in the eigenbasis of $\mathbf{H}$ by the 2 by 1 vector:

$$|\psi(0)\rangle \doteq \begin{bmatrix} \cos\theta/2 \\ \sin\theta/2 \end{bmatrix}.$$

   At later times t, the system is in the state $|\psi(t)\rangle$. Find the 2 by 1 vector that represents $|\psi(t)\rangle$ in the eigenbasis of $\mathbf{H}$.

   (e) Evaluate the time-dependent expectation values in the state $|\psi(t)\rangle$ of the following three observables, where there representation is given in the eigenbasis of $\mathbf{H}$:

      i. $\mathbf{s}_x \doteq \frac{\hbar}{2}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

      ii. $\mathbf{s}_y \doteq \frac{\hbar}{2}\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$

iii. $\mathbf{s}_z \doteq \frac{\hbar}{2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

(f) Evaluate $\langle \mathbf{s}_x \rangle^2 + \langle \mathbf{s}_y \rangle^2 + \langle \mathbf{s}_z \rangle^2$ as a function of time, where the expectation values are in the state $|\psi(t)\rangle$.

(g) Make a sketch of the evolution of the vector $\vec{s}$ as a function of time, where

$$\vec{s} = \langle \mathbf{s}_x \rangle \mathbf{i} + \langle \mathbf{s}_y \rangle \mathbf{j} + \langle \mathbf{s}_z \rangle \mathbf{i}$$

where the expectation values are in the state $|\psi(t)\rangle$. Does the evolution look like the precession of the angular momentum vector of a spinning top?

2. Exercise 5.2.2 on page 163 of your text.

3. Exercise 5.2.3 on page 163 of your text.

4. Exercise 5.2.6 on page 164 of your text. Note that a similar problem was worked out on pages 113-117 of the notes, but there are a few differences; in this problem, the well extends from $x = -a$ to $x = +a$, while the lecture example extended from 0 to L. Also, the potential energy for $x \leq -a$ in this problem is equal to V_0, while the potential energy in the lecture example was equal to ∞ for $x \leq 0$.