Physics 115A Seventh Problem Set

Harry Nelson
Office Hour Tu 2:30-3:30pm, Fr 3:00-4:00pm
TA: Antonio Boveia
Office Hours M 9-10am, Fr 1-3pm PLC
Grader: Victor Soto
Office Hours Th 11:00-12:30pm PLC

due Monday, February 24, 2003

1. $\mathbf{\Omega}$ is a Hermitian operator; show that:

$$(\mathbf{\Delta\Omega})^2 = \langle \mathbf{\Omega}^2 \rangle - \langle \mathbf{\Omega} \rangle^2.$$

   This is not quite the same as Equations 4.2.7 on page 128 of your text.

2. Consider the operator $\mathbf{S}_\eta$, where θ is a real number:

$$\mathbf{S}_\eta \doteq \frac{\hbar}{2} \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}.$$

   (a) Use the characteristic equation to find the eigenvalues of $\mathbf{S}_\eta$.

   (b) Show that the eigenvectors are represented by

$$|{+\hbar/2}\rangle \doteq \begin{bmatrix} \cos(\theta/2) \\ \sin(\theta/2) \end{bmatrix} \qquad |{-\hbar/2}\rangle \doteq \begin{bmatrix} -\sin(\theta/2) \\ \cos(\theta/2) \end{bmatrix}.$$

   It might help to remember that $\cos\theta = \cos^2(\theta/2) - \sin^2(\theta/2)$ and $\sin\theta = 2\sin(\theta/2)\cos(\theta/2)$.

   (c) Consider the initial state:

$$|\psi\rangle \doteq \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

   i. Evaluate $\langle \mathbf{S}_\eta \rangle$, as a function of θ.
   ii. Evaluate $\langle \mathbf{S}_\eta^2 \rangle$, as a function of θ.
   iii. Evaluate $\langle \Delta\mathbf{S}_\eta \rangle$, as a function of θ.

3. Consider the two operators represented by:

$$\mathbf{\Omega} \doteq \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \qquad \mathbf{\Lambda} \doteq \begin{bmatrix} \sqrt{2}b & b & 0 \\ b & \sqrt{2}b & b \\ 0 & b & \sqrt{2}b \end{bmatrix}$$

   (a) Find an orthonormal basis that is an eigenbasis simultaneously of $\mathbf{\Omega}$ and $\mathbf{\Lambda}$; use the **original** representation to describe the eigenbasis.

(b) Do $\mathbf{\Omega}$ and $\mathbf{\Lambda}$ form a complete set of commuting observables?

(c) An initial state is described by:

$$|\psi\rangle \doteq \begin{bmatrix} \sqrt{1/3} \\ \sqrt{1/3} \\ \sqrt{1/3} \end{bmatrix}.$$

First $\mathbf{\Omega}$ is measured, then $\mathbf{\Lambda}$ is measured. Enumerate the sets of eigenvalues that result, and the final states (in the original basis) that correspond to each set of eigenvalues.