Physics 115A Second Problem Set

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Office Hours Tu 2:30-3:30pm
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1. Imagine representing kets $|1\rangle$ and $|2\rangle$ as 3 by 2 matrices, with complex valued entries. That is:
$$|1\rangle \rightarrow \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \\ \alpha_{31} & \alpha_{32} \end{bmatrix} \quad |2\rangle \rightarrow \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \\ \beta_{31} & \beta_{32} \end{bmatrix}.$$
   (a) What matrices represent $\langle 1|$ and $\langle 2|$?
   (b) Evaluate the four inner products, $\langle 1|1\rangle$, $\langle 1|2\rangle$, $\langle 2|1\rangle$, and $\langle 2|2\rangle$; remember to take the trace.
   (c) Would the results to the previous part have been any different if both $|1\rangle$ and $|2\rangle$ had been represented by 6 by 1 column vectors? Take care to discuss how to change the numbering scheme.
2. Are there any vectors in three-dimensional space for which the operation $R(\frac{1}{2}\pi\mathbf{i})R(\frac{1}{2}\pi\mathbf{j})$ yields the same vector as $R(\frac{1}{2}\pi\mathbf{j})R(\frac{1}{2}\pi\mathbf{i})$? If so, explicitly identify at least one such vector.
3. Derive expressions (1.5.10) and (1.5.11) on page 20 of the text.
4. Make the 2 by 2 matrix that represents $|V\rangle\langle V|$, where the ket V is represented by:
$$|V\rangle \rightarrow \begin{bmatrix} e^{i\delta}\sin\theta \\ e^{-i\delta}\cos\theta \end{bmatrix}$$
   and both δ and θ are real numbers. Explicitly evaluate the square of this matrix, to show that it represents a projection operator. Find the general 2 by 1 column vector, that when multiplied by this matrix, gives zero. Does this matrix have an inverse?
5. Exercise 1.6.3, page 28 of the text.