$$1 = |A|^2 \cdot \sqrt{\pi \cdot \Delta^2} \Rightarrow |A| = \frac{1}{(\pi\Delta^2)^{1/4}}$$

and so $\quad \psi(x) = \langle x|\psi\rangle = \dfrac{1}{(\pi\Delta^2)^{1/4}}\, e^{-\frac{(x-a)^2}{2\Delta^2}}$

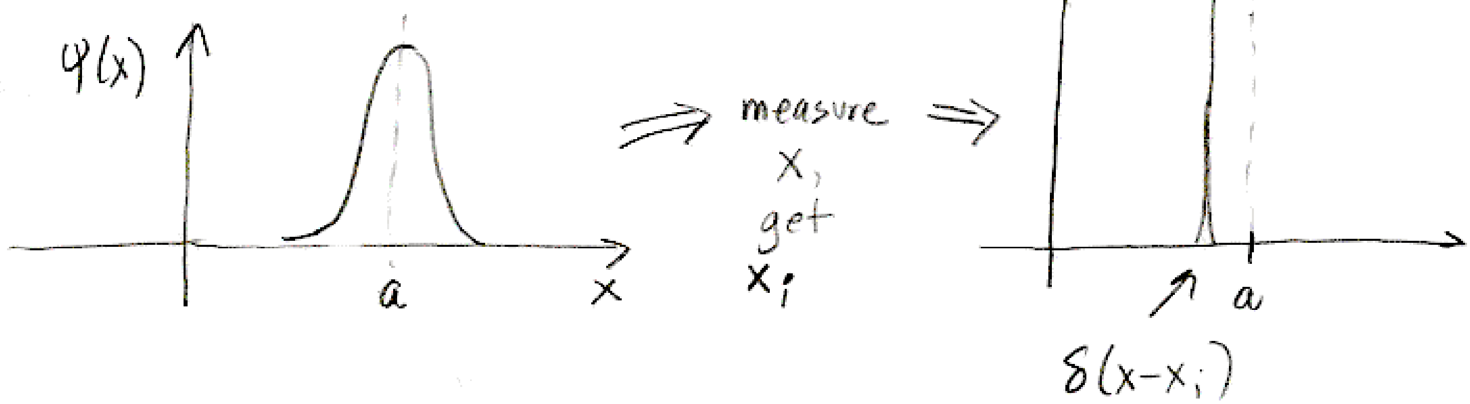
Probability of finding particle between x and $x+dx$ is:

$$P(x)dx = |\psi(x)|^2 dx = \frac{1}{(\pi\Delta^2)^{1/2}}\, e^{-\frac{(x-a)^2}{\Delta^2}}\, dx$$

This still peaks at $x=a$, but now the FWHM is ~~bigger~~ smaller... $\quad \text{FWHM} = \frac{1}{\sqrt{2}} \cdot 2.35 \cdot \Delta$

$$= \cancel{4\sqrt{\ln 2}}\,\Delta = \cancel{3.33}\,\Delta$$
$$\sqrt{4\ln 2}\,\Delta = 1.67\,\Delta$$

Think of measurement of $\underset{\sim}{x}$:



The average, or expectation value, of x will be

$$\langle \underset{\sim}{x} \rangle = \lim_{N\to\infty} \frac{1}{N}\sum_{i=1}^{N} x_i \quad \left(\text{from } N \text{ measurements}\right)$$

$$= \langle\psi|\underset{\sim}{x}|\psi\rangle = \int_{-\infty}^{\infty} \langle\psi|x\rangle dx \langle x|\underset{\sim}{x}|\psi\rangle$$

$$\langle x|\underset{\sim}{x}|\psi\rangle = \int_{-\infty}^{\infty} \langle x|\underset{\sim}{x}|x'\rangle dx' \langle x'|\psi\rangle = \int_{-\infty}^{\infty} x\,\delta(x-x')\,dx'\,\psi(x') = x\,\psi(x)$$