

... origin of the Heisenberg uncertainty principle...

Example

$$\underline{\Omega} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \underline{\Lambda} = \begin{pmatrix} a & b & d \\ b^* & c & b^* \\ d & b & a \end{pmatrix}$$

(unitary...)
aka, "symmetry operator"

$$\underline{\Omega} \underline{\Lambda} - \underline{\Lambda} \underline{\Omega} \stackrel{?}{=} 0$$

$$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} a & b & d \\ b^* & c & b^* \\ d & b & a \end{pmatrix} = \begin{pmatrix} d & b & a \\ b^* & c & b^* \\ a & b & d \end{pmatrix}$$

$$\begin{pmatrix} a & b & d \\ b^* & c & b^* \\ d & b & a \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} d & b & a \\ b^* & c & b^* \\ a & b & d \end{pmatrix}$$

but note: $\underline{\Omega} \underline{\Lambda} - \underline{\Lambda} \underline{\Omega} = 0$

$$\underline{\Lambda} - \underline{\Omega}^{-1} \underline{\Lambda} \underline{\Omega} = 0$$

$$\underline{\Omega}^{-1} = \underline{\Omega}^\dagger \text{ (Unitary)}$$

$$\text{or } \underline{\Omega}^\dagger \underline{\Lambda} \underline{\Omega} = \underline{\Lambda}$$

← $\underline{\Lambda}$ is "invariant" under $\underline{\Omega}$.

$\underline{\Omega}$ is a "symmetry operator"

$$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = 1 \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

obvious eigenvector.