

Contents

1. Mathematical Introduction	1
1.1. Linear Vector Spaces: Basics	1
1.2. Inner Product Spaces	7
1.3. Dual Spaces and the Dirac Notation	11
1.4. Subspaces	17
1.5. Linear Operators	18
1.6. Matrix Elements of Linear Operators	20
1.7. Active and Passive Transformations	29
1.8. The Eigenvalue Problem	30
1.9. Functions of Operators and Related Concepts	54
1.10. Generalization to Infinite Dimensions	57
2. Review of Classical Mechanics	75
2.1. The Principle of Least Action and Lagrangian Mechanics	78
2.2. The Electromagnetic Lagrangian	83
2.3. The Two-Body Problem	85
2.4. How Smart Is a Particle?	86
2.5. The Hamiltonian Formalism	86
2.6. The Electromagnetic Force in the Hamiltonian Scheme	90
2.7. Cyclic Coordinates, Poisson Brackets, and Canonical Transformations	91
2.8. Symmetries and Their Consequences	98
3. All Is Not Well with Classical Mechanics	107
3.1. Particles and Waves in Classical Physics	107
3.2. An Experiment with Waves and Particles (Classical)	108
3.3. The Double-Slit Experiment with Light	110
3.4. Matter Waves (de Broglie Waves)	112
3.5. Conclusions	112

4. The Postulates—a General Discussion	115
4.1. The Postulates	115
4.2. Discussion of Postulates I–III	116
4.3. The Schrödinger Equation (Dotting Your i 's and Crossing your \hbar 's)	143
5. Simple Problems in One Dimension	151
5.1. The Free Particle	151
5.2. The Particle in a Box	157
5.3. The Continuity Equation for Probability	164
5.4. The Single-Step Potential: a Problem in Scattering	167
5.5. The Double-Slit Experiment	175
5.6. Some Theorems	176
6. The Classical Limit	179
7. The Harmonic Oscillator	185
7.1. Why Study the Harmonic Oscillator?	185
7.2. Review of the Classical Oscillator	188
7.3. Quantization of the Oscillator (Coordinate Basis)	189
7.4. The Oscillator in the Energy Basis	202
7.5. Passage from the Energy Basis to the X Basis	216
8. The Path Integral Formulation of Quantum Theory	223
8.1. The Path Integral Recipe	223
8.2. Analysis of the Recipe	224
8.3. An Approximation to $U(t)$ for the Free Particle	225
8.4. Path Integral Evaluation of the Free-Particle Propagator	226
8.5. Equivalence to the Schrödinger Equation	229
8.6. Potentials of the Form $V = a + bx + cx^2 + d\dot{x} + e\dot{x}\dot{x}$	231
9. The Heisenberg Uncertainty Relations	237
9.1. Introduction	237
9.2. Derivation of the Uncertainty Relations	237
9.3. The Minimum Uncertainty Packet	239
9.4. Applications of the Uncertainty Principle	241
9.5. The Energy–Time Uncertainty Relation	245
10. Systems with N Degrees of Freedom	247
10.1. N Particles in One Dimension	247
10.2. More Particles in More Dimensions	259
10.3. Identical Particles	260

11. Symmetries and Their Consequences	279
11.1. Overview	279
11.2. Translational Invariance in Quantum Theory	279
11.3. Time Translational Invariance	294
11.4. Parity Invariance	297
11.5. Time-Reversal Symmetry	301
12. Rotational Invariance and Angular Momentum	305
12.1. Translations in Two Dimensions	305
12.2. Rotations in Two Dimensions	306
12.3. The Eigenvalue Problem of L_z	313
12.4. Angular Momentum in Three Dimensions	318
12.5. The Eigenvalue Problem of L^2 and L_z	321
12.6. Solution of Rotationally Invariant Problems	339
13. The Hydrogen Atom	353
13.1. The Eigenvalue Problem	353
13.2. The Degeneracy of the Hydrogen Spectrum	359
13.3. Numerical Estimates and Comparison with Experiment	361
13.4. Multielectron Atoms and the Periodic Table	369
14. Spin	373
14.1. Introduction	373
14.2. What is the Nature of Spin?	373
14.3. Kinematics of Spin	374
14.4. Spin Dynamics	385
14.5. Return of Orbital Degrees of Freedom	397
15. Addition of Angular Momenta	403
15.1. A Simple Example	403
15.2. The General Problem	408
15.3. Irreducible Tensor Operators	416
15.4. Explanation of Some "Accidental" Degeneracies	421
16. Variational and WKB Methods	429
16.1. The Variational Method	429
16.2. The Wentzel-Kramers-Brillouin Method	435
17. Time-Independent Perturbation Theory	451
17.1. The Formalism	451
17.2. Some Examples	454
17.3. Degenerate Perturbation Theory	464

18. Time-Dependent Perturbation Theory	473
18.1. The Problem	473
18.2. First-Order Perturbation Theory	474
18.3. Higher Orders in Perturbation Theory	484
18.4. A General Discussion of Electromagnetic Interactions	492
18.5. Interaction of Atoms with Electromagnetic Radiation	499
19. Scattering Theory	523
19.1. Introduction	523
19.2. Recapitulation of One-Dimensional Scattering and Overview	524
19.3. The Born Approximation (Time-Dependent Description)	529
19.4. Born Again (The Time-Independent Approximation)	534
19.5. The Partial Wave Expansion	545
19.6. Two-Particle Scattering	555
20. The Dirac Equation	563
20.1. The Free-Particle Dirac Equation	563
20.2. Electromagnetic Interaction of the Dirac Particle	566
20.3. More on Relativistic Quantum Mechanics	574
21. Path Integrals—II	581
21.1. Derivation of the Path Integral	582
21.2. Imaginary Time Formalism	613
21.3. Spin and Fermion Path Integrals	636
21.4. Summary	652
Appendix	655
A.1. Matrix Inversion	655
A.2. Gaussian Integrals	659
A.3. Complex Numbers	660
A.4. The $i\epsilon$ Prescription	661
ANSWERS TO SELECTED EXERCISES	665
TABLE OF CONSTANTS	669
INDEX	671