Some quantum mechanics, and k

plane wave

wave $\sim \cos \left(2\pi \frac{x}{\lambda} - 2\pi \frac{t}{T} \right)$

wavelength $\lambda$, period $T$

define: $k = \frac{2\pi}{\lambda}$, $w = \frac{2\pi}{T}$

wave $\sim \Re \left( e^{i(kx - wt)} \right)$ or $e^{i(kx - wt)}$

Note: suppose one slit is closed.....

at "central max", $I_{\text{both}} \approx 4I_{\text{top}} = 4I_{\text{bottom}}$

and $I_{\text{both}} \neq I_{\text{top}} + I_{\text{bottom}}$

because light is waves

bullets: $I_{\text{both would}} = I_{\text{top}} + I_{\text{bottom}}$
electrons are waves too!

\[ \lambda_{\text{electron}} = \frac{\lambda_D}{e} \]

\[ \lambda_{\text{visible light}} \ll \lambda_{\text{electron}} \]

\[ I_{av} \]

Because the wavelength of an electron is so small, it is hard in an experiment to resolve the maxima. The "average" usually measured.

Actually, "looks" like \[ I_{av} = I_{top} + I_{bottom} \]

\[ \lambda_{\text{electron}} = \frac{h}{p} \leq \text{planck's constant} \]

or, \[ k_{\text{elec}} = \frac{2\pi}{\lambda_{\text{elec}}} = \frac{2\pi}{(h/p)} = \left( \frac{2\pi}{h} \right) p = \frac{p}{\hbar} \]

where \[ \hbar = \hbar/2\pi \]

**Time dependence:** \[ w = \frac{E}{\hbar} \]

How can something be both a particle and a wave? Later
Representation of $k$ in coordinate space.

\[ x |x\rangle = x |x\rangle \quad k |k\rangle = k |k\rangle \]

$(k$ is an eigenket of $k$ operator$)$

where, to get from one rep to other, use unitary transformations

\[ \langle k | x \rangle = \frac{1}{\sqrt{2\pi}} e^{ikx} \quad \langle x | k \rangle = \frac{1}{\sqrt{2\pi}} e^{-ikx} \]

Can easily represent $\hat{x}$ and $\hat{k}$ in the basis of their eigenkets; they are Hermitean.

\[ \langle x | \hat{x} | x \rangle = x \delta(x-x') \quad \langle k | \hat{k} | k \rangle = k \delta(k-k') \]

\[ = x' \delta(x-x') \quad = k' \delta(k-k') \]

How do these operators look in each other's bases?

\[ \langle x | k | x' \rangle = \int dk' dk \langle x | k' \rangle \langle k' | \hat{k} | k \rangle \langle k | x' \rangle \]

**Integrate over $k'$**

\[ = \int dk \langle x | k \rangle k \langle k | x \rangle \]

\[ \langle x | \hat{k} | x \rangle = \frac{1}{2\pi} \int dk k e^{ik(x-x')} = \frac{i}{\pi x} \int dk e^{ik(x-x')} \]

\[ \int_{-\infty}^{\infty} dk e^{ik(x-x')} \]
\[ \langle x \mid k \mid x' \rangle = \frac{i}{\hbar} \frac{d}{dx} \delta(x - x') = \frac{i}{\hbar} \delta'(x - x') \]

what does \( k \) do to \( f \)?

\[ \langle x \mid k \mid f \rangle \rightarrow \text{ what function is } \langle x \mid k \mid f \rangle? \]

\[ \langle x \mid k \mid f \rangle = \int dx' \langle x \mid k \mid x' \rangle \times \langle x' \mid f \rangle = \frac{1}{i} \frac{df}{dx} \]

\[ = \frac{1}{i} \delta'(x - x') f(x) \]

\[ \text{use 1.10, 21 p.62} \]

"\( k \) is not diagonal"

that is,

\[ \langle x \mid k \mid f \rangle \equiv \frac{1}{i} \frac{df}{dx} \]

sometimes said.....

\[ \langle k \mid f \rangle = \frac{1}{i} \frac{df}{dx} \]

or \( k \times f(x) = \frac{1}{i} \frac{df}{dx} \)

note that \( \langle k \mid (f(x) g(x)) \rangle = \frac{1}{i} \left( \frac{df}{dx} g + f \frac{dg}{dx} \right) \)

what is \( [k, x] \)?

\[ \langle k \times x \rangle f(x) = k \times f(x) = \frac{1}{i} \frac{df(x)}{dx} \]
\[
\begin{align*}
= \frac{1}{i} \frac{d}{dx} (xf(x)) - x \frac{1}{i} \frac{d}{dx} \\
= \frac{1}{i} f(x) + \frac{1}{i} x \frac{df}{dx} - \frac{1}{i} x \frac{d}{dx}
\end{align*}
\]

\[
\left[ \frac{1}{i}, x \right] f(x) = \frac{1}{i} f(x)
\]
or

\[
\left[ \frac{1}{i}, x \right] = \frac{1}{i} (1, 10, 41) p. 70
\]

Is \( \frac{1}{i} \) Hermitian? \( \text{in } k \)-space, obviously yes \( \text{ (eigenvalues) } \) \( \text{real} \).

In \( x \)-space?

If so, \( \langle g \mid \frac{1}{i} \mid f \rangle = \langle f \mid \frac{1}{i} \mid g \rangle^* \)

\[
\int \int dx' dx \left\langle g \mid x \right\rangle \left\langle x \left| \frac{1}{i} \right| x' \right\rangle \left\langle x' \left| f \right\rangle \right. \overset{?}{=} \int \int dx' dx \left\langle f \left| \frac{1}{i} \right| x \right\rangle \left\langle x \left| g \right\rangle \right. \\
\left. \frac{1}{i} \delta'(x-x') f(x') \right.
\]

\text{general case } \begin{cases} \text{(Do } x' \text{ integral)} \\
\int_a^b dx g^*(x) \frac{df(x)}{dx} = \int_a^b dx f(x) \frac{dg^*(x)}{dx} \\
\text{integrate by parts... } = \left. \left[ \frac{1}{i} g^*(x)f(x) \right] \right|_a^b - \frac{1}{i} \int_a^b dx f(x) \frac{dg^*(x)}{dx}
\end{cases}

\( k \) is Hermitian

when
\[
\frac{1}{i} \int_{a}^{b} g^*(x) f(x) dx = 0 \quad \text{"surface term"}
\]

- very important
- actually,
- as long as we work in a subspace where all kets satisfy the vanishing of the surface term.

New Concept

subspace where \( k \) is Hermitian.

space of all kets

(1.10.29, p. 65)