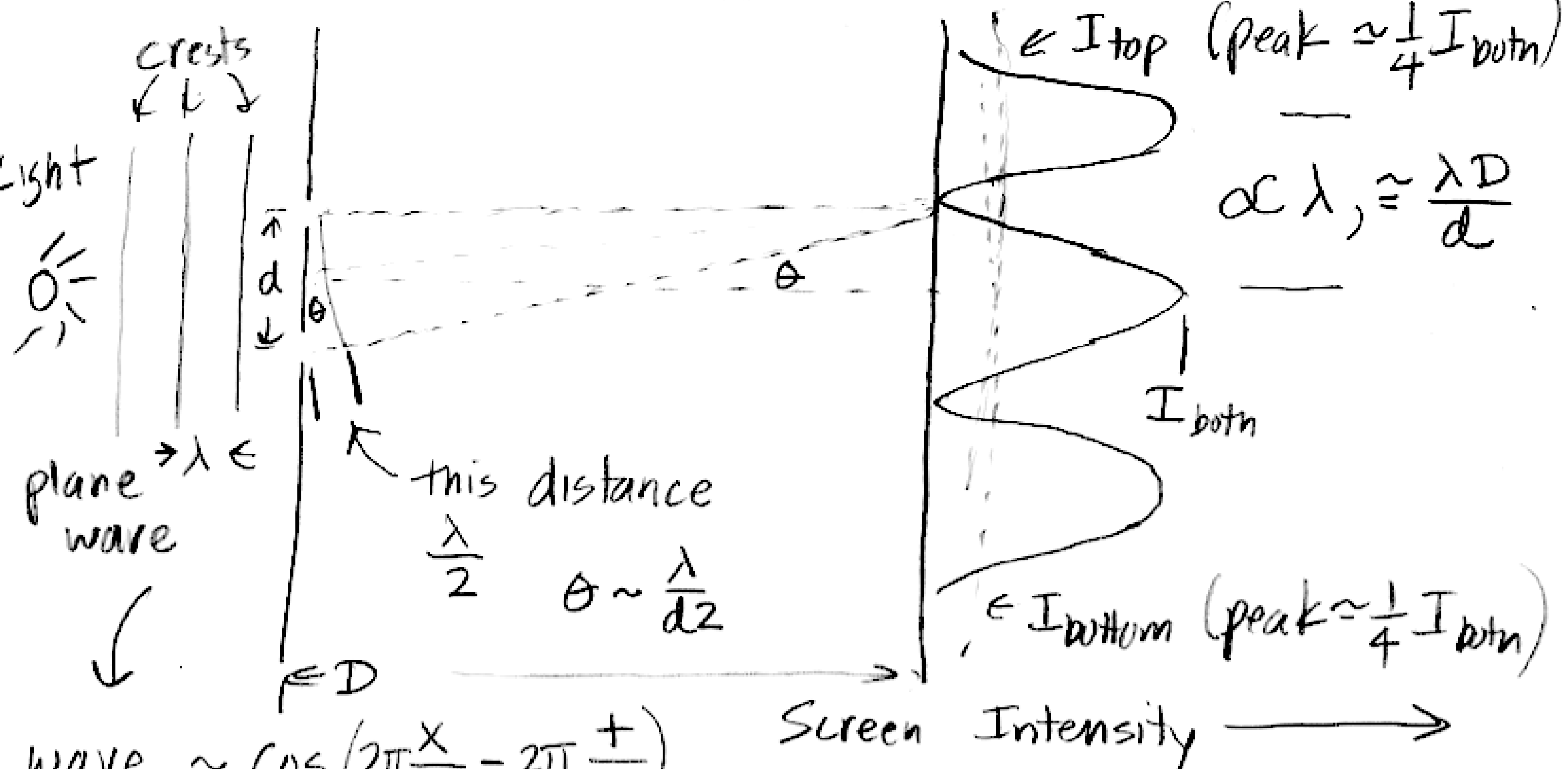
Some quantum mechanics, and k



wave $\sim \cos\left(2\pi \frac{x}{\lambda} - 2\pi \frac{t}{T}\right)$

wavelength (λ), period (T)

define: $k \equiv \frac{2\pi}{\lambda}$ $\quad \omega \equiv \frac{2\pi}{T}$

wave $\sim \mathrm{Re}\left(e^{i(kx-\omega t)}\right)$ or $e^{i(kx-\omega t)}$ (take real part in the end, if necessary)

Note: suppose one slit is closed....

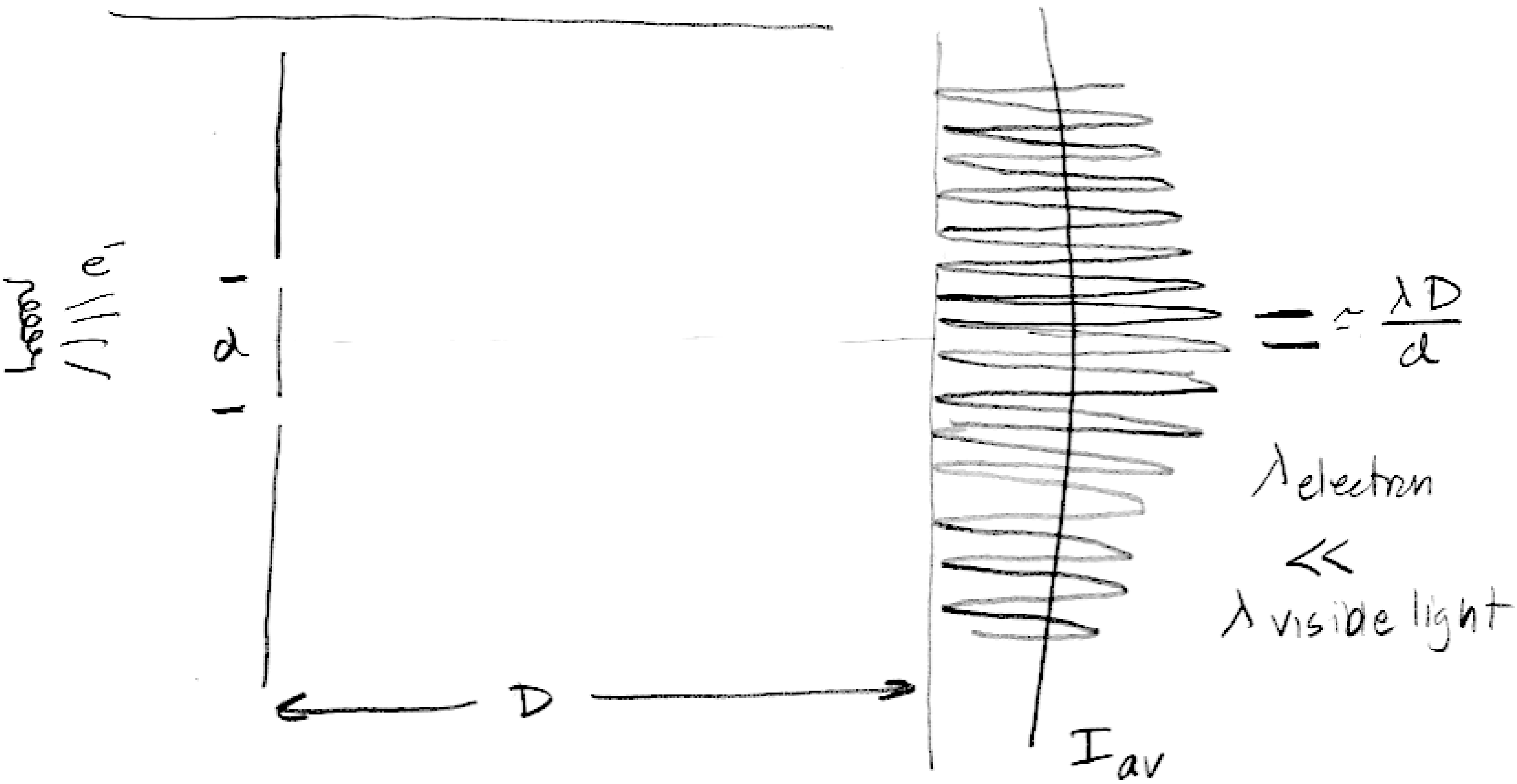
at "central max", $I_{both} \simeq 4 I_{top} \simeq 4 I_{bottom}$

and $I_{both} \neq I_{top} + I_{bottom}$

because light is <u>waves</u>

bullets: I_{both} would $= I_{top} + I_{bottom}$

electrons are waves too!



Because the wavelength of an electron is so small, it is hard in an experiment to resolve the maxima. The "average" usually measured.

Actually, "looks" like $I_{av} = I_{top} + I_{bottom}$

$$\lambda_{electron} = \frac{h}{p} \quad \begin{array}{l} \leftarrow \text{planck's constant} \\ \leftarrow \text{momentum of electron} \end{array}$$

$$\text{or, } k_{elect} = \frac{2\pi}{\lambda_{elect}} = \frac{2\pi}{(h/p)} = \left(\frac{2\pi}{h}\right) p = \frac{p}{\hbar}$$

$$\text{where} \quad \hbar \equiv h/2\pi$$

time dependence: $\quad \omega = \frac{E}{\hbar}$

How can something be both a particle an a wave? Later

Representation of $\underset{\sim}{k}$ in coordinate space

$\underset{\sim}{x}|x\rangle = x|x\rangle$ $\qquad$ $\underset{\sim}{k}|k\rangle = k|k\rangle$

(k is an eigenket of $\underset{\sim}{k}$ operator)

where, to get from one rep to other, use unitary transformations

$$\langle k|x\rangle = \frac{1}{\sqrt{2\pi}} e^{-ikx} \qquad \langle x|k\rangle = \frac{1}{\sqrt{2\pi}} e^{ikx}$$

Can easily represent $\underset{\sim}{x}$ and $\underset{\sim}{k}$ in the basis of their eigenkets; they are Hermitian.

$$\langle x|\underset{\sim}{x}|x'\rangle = x\,\delta(x-x') \qquad \langle k'|\underset{\sim}{k}|k\rangle = k\,\delta(k-k')$$
$$= x'\,\delta(x-x') \qquad\qquad = k'\,\delta(k-k')$$

How do these operators look in each other's bases?

$$\langle x|\underset{\sim}{k}|x'\rangle = \int_{-\infty}^{\infty}\!\!\int_{-\infty}^{\infty} dk'\,dk\,\langle x|k'\rangle \underbrace{\langle k'|\underset{\sim}{k}|k\rangle}_{k\,\delta(k-k')} \langle k|x'\rangle$$

, integrate over k'

$$= \int_{-\infty}^{\infty} dk\,\langle x|k\rangle\, k\,\langle k|x'\rangle$$

$$\langle x|\underset{\sim}{k}|x'\rangle = \frac{1}{2\pi}\int_{-\infty}^{\infty} dk\, k\, e^{ik(x-x')} = \frac{1}{i}\frac{d}{dx}\left[\frac{1}{2\pi}\int_{-\infty}^{\infty} dk\, e^{ik(x-x')}\right]$$

$$\langle x | \underset{\sim}{k} | x' \rangle = \frac{1}{i} \frac{d}{dx} \delta(x - x') = \frac{1}{i} \delta'(x-x')$$

what does $\underset{\sim}{k}$ do to f?

$\underset{\sim}{k} | f \rangle \rightarrow$ what function is $\langle x | \underset{\sim}{k} | f \rangle$?

$$\langle x | \underset{\sim}{k} | f \rangle = \int dx' \underbrace{\langle x | \underset{\sim}{k} | x' \rangle}_{\frac{1}{i}\delta'(x-x')} \underbrace{\langle x' | f \rangle}_{f(x')} = \frac{1}{i} \frac{df}{dx}$$

insert unity

use 1.10,21 p.62

that is, $\underset{\sim}{k} | f \rangle \doteq \frac{1}{i} \frac{df}{dx}$

"$\underset{\sim}{k}$ is not diagonal"

sometimes said.....

$$\underset{\sim}{k} | f \rangle = \left| \frac{1}{i} \frac{df}{dx} \right\rangle$$

or $\underset{\sim}{k} f(x) = \frac{1}{i} \frac{df}{dx}$

note that $\underset{\sim}{k} (f(x) g(x)) = \frac{1}{i} \left(\frac{df}{dx} g + f \frac{dg}{dx} \right)$

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what is $[\underset{\sim}{k}, \underset{\sim}{x}]$?

$$(\underset{\sim}{k}\underset{\sim}{x} - \underset{\sim}{x}\underset{\sim}{k}) f(x) = \underset{\sim}{k}\, \underline{x f(x)} - \underset{\sim}{x} \frac{1}{i} \frac{df(x)}{dx}$$

$$= \frac{1}{i}\frac{d}{dx}(x f(x)) - x\frac{1}{i}\frac{df}{dx}$$

$$= \frac{1}{i} f(x) + \cancel{\frac{1}{i} x \frac{df}{dx}} - \cancel{\frac{1}{i} x \frac{df}{dx}}$$

$$[\underset{\sim}{k}, \underset{\sim}{x}]\, f(x) = \frac{1}{i} f(x)$$

or $$[\underset{\sim}{k}, \underset{\sim}{x}] = \frac{1}{i} \qquad (1,10,41)\ \text{p. }70$$

<u>Is $\underset{\sim}{k}$ Hermitian?</u> in k-space, obviously <u>yes</u> (eigenvalues real).

In x-space?

if so, $\langle g | \underset{\sim}{k} | f \rangle = \langle f | \underset{\sim}{k} | g \rangle^*$

$$\iint dx'\,dx\, \underbrace{\langle g | x \rangle}_{g^*(x)} \underbrace{\langle x | \underset{\sim}{k} | x' \rangle}_{\frac{1}{i}\delta'(x-x')} \underbrace{\langle x' | f \rangle}_{f(x')} \overset{?}{=} \iint dx'\,dx\, \underbrace{\langle f | x \rangle^*}_{f(x)} \underbrace{\langle x | \underset{\sim}{k} | x' \rangle^*}_{-\frac{1}{i}\delta'(x-x')} \underbrace{\langle x' | g \rangle^*}_{g^*(x')}$$

- Do x' integral

general case >

$$\underbrace{\frac{1}{i}\int_a^b dx\, g^*(x)\frac{df(x)}{dx}}_{\text{integrate by parts}} \overset{?}{=} -\frac{1}{i}\int_a^b dx\, f(x)\frac{dg^*}{dx}(x)$$

integrate by parts ... $= \frac{1}{i} g^*(x) f(x)\Big|_a^b - \frac{1}{i}\int_a^b dx\, f(x)\frac{dg^*}{dx}(x)$

$\underset{\sim}{k}$ is Hermitian

when $\frac{1}{i} g^*(x) f(x) \Big|_a^b = 0$ "surface term"

(1.10.29, p.65)

- very important actually.
- as long as we work in a subspace where all kets satisfy the vanishing of the surface term.

<u>New Concept</u>: