(1) Gravity is balanced by the force due to the expulsion of the gas from the rockets

$$m g_m = -\frac{dp}{dt} = -v\frac{dm}{dt}$$

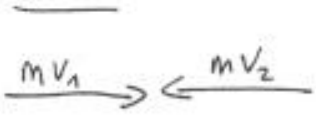
$$g_m\, dt = -v\frac{dm}{m}$$

$$g_m t = -v \log\frac{m}{m_0}$$

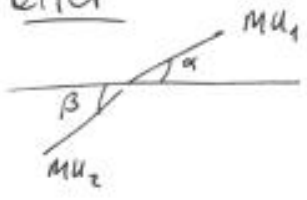
$$t = -\frac{v}{g_m}\log\frac{2}{3} = \log\frac{3}{2}\left(\frac{v}{g_m}\right)$$

(2)

before



after



NB: $\alpha \neq \beta$!

Conservation of momentum

$$mv_1 - mv_2 = mu_1\cos\alpha - mu_2\cos\beta \quad (1)$$

$$0 = mu_1\sin\alpha - mu_2\sin\beta \quad (2)$$

Conservation of energy

$$\frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 = \frac{1}{2}mu_1^2 + \frac{1}{2}mu_2^2 \quad (3)$$

NB: m cancels from equations (1) (2) & (3) ....